International Journal of Solids and Structures 51 (2014) 886–893

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Crack problem for superconducting strip with finite thickness

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ARTICLE INFO

Article history:
Received 20 March 2013
Received in revised form 10 November 2013
Available online 23 November 2013

Keywords:
Superconducting strip
Inclined crack
Variational inequality
Stress intensity factor

ABSTRACT

The inclined crack problems are considered for a thin strip and a strip with finite thickness in a perpendicular magnetic field. The critical current density is assumed to be a constant. The crack orientation is varied and the effect of crack on the magnetic field distribution is neglected. Based on the analytical results and variational inequality, the field and current distributions are computed for both thin strip and strip with finite thickness cases, respectively. Then, the stress intensity factors at the crack tip are determined using the finite element method for magnetic field loads. The numerical results are presented for different inclined crack angles, magnetization processes and geometry parameters of the strip. The results show that the fracture behavior of the strip with finite thickness is more complicated than that of the thin strip. With the numerical results, we can predict the largest possibility of cracking as the strip is in an external field.

1. Introduction

Superconductivity is a phenomenon of zero electrical resistance below a certain temperature. The critical current density is one of the most important characteristics of the superconductors, which represents the maximum current density that can be carried in a superconductor. In addition, high magnetic field can be trapped in the superconductors, and trapped field is dependent on the critical current density. Due to their unique properties of high critical current density and trapped field, high temperature superconductors (HTS) may be widely applied in many applications, such as magnetic separation, motor, generator and magnetron spattering (Oka et al., 2012; Yokoyama et al., 2011). Recently, with the development of the superconductors of strong flux pinning properties, high critical current were achieved. At liquid nitrogen temperature, YBCO on metal substrates could reach current values in excess of 1 MA cm^{-2} (Foltyn et al., 1999). The critical current density J_c of very thin YBCO films can approach 10 MA cm^{-2} at 77 K in self field (Foltyn et al., 2009).

Although the superconductors have remarkable properties, the mechanical behavior of the superconductor has been paid much attention in recent years (Johansen, 2000). A major problem is the brittleness of the superconductors. Application of the magnetic field will induce the shielding current. As the magnetic field is smaller than the lower critical field B_{c1}, the shielding current only flows near the surfaces. When the magnetic field becomes large enough, the magnetic flux and shielding current begin to penetrate into the sample. Then, due to the effects of defects and flux pinning, the electromagnetic body force (Lorentz force) is generated by the interaction of the magnetic field and current. Loaded by the electromagnetic force, the superconductor will undergo magnetostriction. With the increasing of the critical current density and applied magnetic field, the stress induced by the electromagnetic body force may be greater than critical value and lead to the mechanical failure of the superconductors. Ikuta et al. (1993) studied a large magnetostriction in a single crystal firstly, and a quantitative model was proposed. Subsequently, the magnetostriction was investigated based on three critical state models (Ikuta et al., 1994). The pinning induced magnetostriction in an isotropic superconductor was calculated for different sample shapes, such as an infinite slab (Ikuta et al., 1993), a rectangular slab (Johansen, 1999b), a square cylinder (Johansen et al., 1998) and a circular cylinder (Johansen et al., 1995; Johansen, 1999a), a hollow cylinder and a clamped cylinder (Johansen et al., 2000, 2001), a superconducting thin circular disk (Johansen and Shantsev, 2003), the full magnetization cycle and virgin branch in thin long strip (Eremenko et al., 1998; Nabialek et al., 1998). Eremenko et al. (1998) also performed measurements for a quantitative comparison, and the results showed that measured and calculated curves have similar features and close magnitudes of deformation. The flux pinning induced magnetostriction was studied experimentally for different temperatures (Nabialek et al., 1997). The contributions of Meissner current and normal state to the magnetostriction were also investigated (Celebi et al., 2005, 2007). The stress distributions were calculated for a slab based on the critical state Kim model and a superconducting strip with transport current (Yong and Zhou,

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0020-7683/$ - see front matter
http://dx.doi.org/10.1016/j.ijsolstr.2013.11.013

2008, 2011b). Feng et al. (2011) investigated the stress and magnetostriction in the functionally graded slab.

In addition, cracks and defects may be found during fabrication and low temperature conditions (Diko and Krabbes, 2003; Katagiri et al., 2008). As the superconductors are subjected to high magnetic field in service, the initiation and propagation of crack may result in the failure of these materials. Ren et al. (1995) observed cracking in a mini-magnet activated by an applied field of 14 T. The cracking of the samples will limit the trapped field of bulk superconductors at lower temperature (Fuchs et al., 2000; Tomita and Murakami, 2003). Zhou and Yong (2007) analyzed the central crack problem for the long rectangular slab under the electromagnetic force. It was found that the stress intensity factor can be reduced by decreasing the maximum field. The central crack, interface crack and two collinear cracks problems in the thin strip, and the crack problem in thin strip by considering the decreasing of critical current with thickness were also investigated (He et al., 2013; Yong and Zhou, 2011a, 2012; Yong et al., 2013). The shear and transverse stress distributions in coated conductors were analyzed (Jing et al., 2013). The interaction of two collinear cracks and crack-inclusion problems in a rectangular slab were studied (Gao et al., 2010a,b). The fracture behavior for an inhomogeneous orthotropic superconducting slab was investigated by Feng et al. (2012). The inclined crack problem in the rectangular slab for different magnetization processes was also studied (Wang et al., 2013).

In spite of all the extensive theoretical work done on the fracture behavior, the crack problem in the strip of finite thickness has not been studied. In this paper, the general problems of an inclined crack in a superconducting thin strip and a strip with finite thickness are studied. Since the thickness is assumed to be half of the width, the strip with finite thickness can be regarded as a bulk. The state of deformation is assumed to be plane strain. After we present the electromagnetic behavior in the strip, the stress intensity factors are obtained numerically. Both the increasing field case and decreasing field case are considered. The effects of the geometry parameters of the strip and the orientations of crack on the stress intensity factors are discussed.

2. Thin strip in a perpendicular field

Consider a long superconducting thin strip of width $2a$ and thickness $2b$ which is attached to a substrate, as shown in Fig. 1(a). The strip is infinitely long in the y direction and assumed to be isotropic. An inclined crack of length $2c$ is in the strip and the crack is located at the center (Fig. 1(b)). The inclined angle is θ, and the crack can be oriented from a parallel position ($\theta = 0$) to a perpendicular position ($\theta = \pi/2$). A uniform magnetic field B_a is applied parallel to the z axis. Due to the symmetry of the problem, the shielding current inside the superconductor is only along the y direction. In addition, the shielding current induced by the magnetic field screens the interior magnetic field of the superconductor. Since the shielding current flows in a pattern of rectangular loop, the direction of the shielding current is opposite for the left and right parts of the strip and the current density J has opposite sign. When the superconductor is sufficiently long in the applied field direction, the demagnetization effect is negligible. However, strong demagnetization effect will occur in the strip geometry. For simplicity, we neglect the effect of the crack on the magnetic field distribution. Under this assumption the sample will contain the symmetric flux and current distributions.

By ignoring the effect of the lower critical field B_{c1}, we can obtain the flux density and current density distributions for the magnetized state. Due to the interaction of the magnetic field and current, the superconductor is subjected to the electromagnetic body force and shows a giant magnetostriction. Furthermore, the local loadings for the vertical crack and inclined crack are different (Fig. 1(c)). In the following part, we will consider the zero-field cooling condition. The strip is cooled in zero field, then the magnetic field B_a is applied. In order to obtain the mechanical behavior of the superconductor, it is necessary to determine the flux profiles and current density firstly.

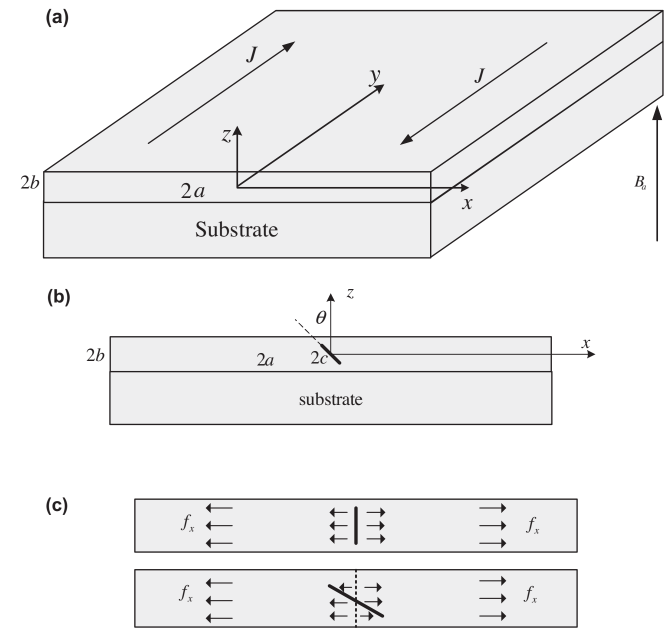

Fig. 1. (a) Schematic of the superconducting strip with substrate. An external field is applied perpendicular to the strip. (b) Crack geometry in the strip. (c) The loadings in thin strip for vertical crack and inclined crack.

2.1. Increasing field case

Start from the exact expressions of the shielding current distribution for a thin superconducting strip in perpendicular magnetic field. For the Bean model, the shielding current density J in the flux penetration region is equal to the critical current density J_c and independent of the magnetic field. In the center of the strip where the magnetic field is screened and $B_z = 0$, the shielding current density is less than J_c. When the applied field increases from the virgin state, one can obtain the following current profiles with conformal transformation (Brandt and Indenbom, 1993; Eremenko et al., 1998)

$$J(x) = \frac{2J_c}{\pi} \arctan \sqrt{\frac{1-(a_0/a)^2}{(a_0/x)^2 - 1}}, \quad |x| < a_0 \tag{1}$$

$$J(x) = J_c x/|x|, \quad a_0 < |x| < a \tag{2}$$

where the penetrating flux front $a_0 = a/\cosh(B_a/B_0)$ and the characteristic field $B_0 = 2\mu_0 J_c b/\pi$. By using the Biot-Savart law, the magnetic field is $B_z(x) = 0$, $|x| < a_0$ and $B_z(x) = \mu_0 \frac{2J_c b}{\pi} \operatorname{arctanh}\left[\sqrt{\frac{1-a_0^2/x^2}{1-a_0^2/a^2}}\right]$, $a_0 < |x| < a$ (Brandt and Indenbom, 1993). It is to be noted that for the thin strip where $b \ll a$, the current density and the magnetic field are averaged over the thickness. In addition, the magnetic field is only along the z direction and $B_x = 0$. Both the current and magnetic field can be considered as scalar quantities. The body force i.e. electromagnetic force $\vec{f} = \vec{J} \times \vec{B}$ is only along the x direction. Furthermore, due to the reason that the strip is infinite along the y direction, we can treat this problem as plane strain case.