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Equivalence of the notch stress intensity factor, tip opening displacement and energy release rate for a sharp V-notch

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ABSTRACT

For an infinite elastic plane with a sharp V-notch under the action of symmetrically loading at infinity, the length of crack initiation ahead of the V-notch's tip is estimated according to a modified Griffith approach. Applying a new conservation integral to the perfectly plastic strip (Dugdale model) ahead of the V-notch's tip, the relationship between notch stress intensity factor (NSIF) and notch tip opening displacement (NTOD) is presented. Also, the relationship between NSIF and perfectly plastic strip size (PPSS) is found. Since there are three fracture parameters (NSIF, NTOD, and PPSS) with changeable notch opening angle in two basic relationships, it is necessary to select one critical parameter with changeable notch opening angle or two independent critical parameters, respectively. With the help of a characteristic length, it is found by this new conservation integral that the NSIF, NTOD and energy release rate are equivalent in the case of small-scale yielding. Especially, the characteristic length possesses clear physical meaning and, for example, depends on both the critical NSIF and SIF or both the NTOD and CTOD, respectively, in which SIF and CTOD are from the tip of a crack degenerated from the sharp V-notch. The dependence of NSIF on NTOD and PPSS is presented according to the equivalence, and the critical NSIF depending on the critical NTOD with a notch opening angle is also predicted.

1. Introduction

A review article (Savruk and Kazberuk, 2010) points out that although the methods for cracked bodies are most completed, in the case of bodies with V-notches these methods are at an early stage of development. Indeed, since the intensity of stress field near the tip of a crack can be measured by *J*-integral, it is proved that the SIF, CTOD and energy release rate are equivalent (Rice, 1968a,b). Clearly, the equivalence means a kind of completeness of the theory for a small-scale plastic zone ahead of the tip of a crack.

However, by calculating *J*-integral around the V-notch's tip at $r = 0$, the same usage as that for a crack gives a function of radial r (Lazzarin et al., 2002)

$$J = \frac{\beta_{1r}^2 \sin\gamma}{2\lambda - 1} \frac{(K_I^N)^2}{E'} r^{2\lambda-1}, \quad \left(\frac{1}{2} \le \lambda < 1\right), \tag{1}$$

where K_I^N is the NSIF, λ is the real positive eigenvalue for Mode I problem, and β_{1r} and γ are the parameters that depend on notch opening angles. Here, E' is given by the Young's modulus E for plane stress or $E/(1-\nu^2)$ for plane strain, in which ν is the Poisson's ratio. Although the *J*-integral has been modified for solving various problems (Lazzarin et al., 2002; Livieri, 2008; Livieri and Tovo, 2009), establishing an equivalent identity of the NSIF, energy release rate and NTOD with plasticity for a sharp V-notch is an open problem.

For example, based on finite fracture mechanics approach, a structure parameter for failure of a sharp V-notch has been discussed for establishing brittle fracture criteria (Neuber, 1958; Knesl, 1991; Seweryn, 1994; Lazzarin and Zambardi, 2001; Ayatollahi and Torabi, 2010; Carpinteri et al., 2010). Typically, Carpinteri et al. (2008) presents the values Δ_{SE} of finite crack extension from the tip of a sharp V-notch

$$\Delta_{SE} = \frac{2}{\lambda\psi^2}\left(\frac{K_{IC}}{\sigma_u}\right)^2, \tag{2}$$

where K_{IC} is the fracture toughness of a crack, σ_u is the ultimate tensile strength for brittle materials, and ψ is a parameter depending on notch opening angles. On the other hand, the energy criteria have been developed (Lazzarin and Zambardi, 2001; Yosibash et al., 2004; Lazzarin and Berto, 2005; Berto and Lazzarin, 2009). With the help of the asymptotic matching technique, considering the potential energy change, Leguillon (2002) gave an estimation of crack initiation length c ahead of the V-notch's tip

$$c = \frac{G_C[s_\theta(\theta_0)]^2}{K(\omega,\theta_0)\sigma_C^2}, \tag{3}$$

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where G_C is the critical energy release rate, σ_C is the critical tension, both $s_\theta(\theta_0)$ and $K(\omega,\theta_0)$ are the coefficients depending on crack initiation direction and notch opening angle. Clearly, both expressions (2) and (3) mean that a crack may emanate from the tip of a sharp V-notch in a jump manner suddenly.

The plastic theory has been applied to a sharp-V-notch with experiments (Kuang and Xu, 1986; Stranberg, 2002; Gomez and Elices, 2003). In the respect of small-scale plastic zone ahead of the V-notch's tip, the problem of perfectly plastic strip (Dugdale, 1960) has been solved by Savruk et al. (2003). It is questionable to aforehand assume that the critical NTOD is independent of the opening angle of a sharp V-notch. Making use of the asymptotic matching technique, Henninger et al. (2007) and Murer and Leguillon (2010) analyzed the cohesive zone model ahead of a sharp V-notch. The obtained NTOD contains a reference SIF from a related crack, and this kind of solution seems to be incomplete since the stress state of a perfectly plastic strip itself is independent of a configuration and the reference SIF may not be necessary. Even so, they recognized some facts and indicated necessary for further research.

Recently, according to Noether's theorem (Noether, 1918), a new conservation integral has been proposed (Shi, 2012)

$$\operatorname{Im}\oint_\Gamma \zeta(z)[\phi'(z)]^2 dz = 0, \tag{4}$$

where $\zeta(z)$ is any conformal transformation which can be adjusted, and $\phi(z)$ is an analytic function which may come from the general solution of a plane physics problem. In this paper, based on this conservation integral (4), equivalence of the NSIF, NTOD and energy release rate is proved, from which the open problems mentioned above are inspected.

In Section 2, the energy release rate of an initial crack emanating from the tip of a sharp V-notch is discussed. In Section 3, a perfectly plastic strip (Dugdale model) ahead of the V-notch's tip is considered, and the conservation integral (4) is applied to obtain NTOD. In Section 4, an equivalent identity among the NSIF, NTOD and energy release rate is presented. Based on the equivalence, the critical parameters can be selected appropriately. Concluding remarks is presented in Section 5.

2. Energy release rate

2.1. An initial crack emanating from the tip of a sharp V-notch

As shown in Fig. 1(a), for an infinite elastic plane with a sharp V-notch, if there is no crack, the stress σ_{22} near the V-notch's tip under the action of symmetrically loading at infinity is (Williams, 1952; Gross and Mendelson, 1972)

$$\sigma_{22} = \frac{K_I^N}{\sqrt{2\pi}} x_1^{\lambda-1}, \quad \left(\frac{1}{2} \le \lambda < 1\right), \tag{5a}$$

where the lowest real positive eigenvalue λ with physical meaning comes from an eigen equation

$$\sin(2\lambda\alpha) + \lambda\sin(2\alpha) = 0 \tag{5b}$$

It is assumed that the crack initiation ahead of the tip of a sharp V-notch occurs, as shown in Fig. 1(b). The relationship between V-notch's K_I^N and $K_I(\alpha)$ at the tip $x_1 = c$ of initial crack has been discussed by Hasebe and Iida (1978), Philipps et al. (2008) and Livieri and Tovo (2009). Carpinteri et al. (2008) gave an expression

$$K_I(\alpha) = \frac{H(\alpha)}{\pi} c^{\lambda-1/2} K_I^N, \tag{6a}$$

where c is the length of an initial crack, and

$$H(\alpha) = B\left(\lambda,\frac{1}{2}\right)\sqrt{\frac{\pi(2\alpha+\sin 2\alpha)}{2(\alpha^2-\sin^2\alpha)}} + B\left(\lambda+1,\frac{1}{2}\right)g(\alpha) + B\left(\lambda+2,\frac{1}{2}\right)h(\alpha), \tag{6b}$$

$$g(\alpha) = -1 - 3\sqrt{\frac{\pi(2\alpha+\sin 2\alpha)}{2(\alpha^2-\sin^2\alpha)}} + f_1(\alpha), \tag{6c}$$

$$h(\alpha) = 2 + 2\sqrt{\frac{\pi(2\alpha+\sin 2\alpha)}{2(\alpha^2-\sin^2\alpha)}} - f_1(\alpha), \tag{6d}$$

$$f_1(\alpha) = [1.103 + 3.615(\alpha/\pi)^2 - 0,718(\alpha/\pi)^3]/\sqrt{(\alpha/\pi)^3}, \tag{6e}$$

wherein $B(\lambda,1/2)$, $B(\lambda+1,1/2)$ and $B(\lambda+2,1/2)$ are the Beta function. After checking all the functions in (6b) with the help of (A-2) in Appendix, it is known that

$$H(\alpha) \to H(\pi) = \pi, \tag{7}$$

when the sharp V-notch degenerates into a semi-infinite crack $\beta = \pi - \alpha = 0$ shown in Fig. 1(b).

Near the tip $x_1 = c$ of an initial crack, a displacement jump across upper and lower surfaces is (Rice, 1968b)

$$\Delta u_2 = u_2^+ - u_2^- = \frac{8K_I(\alpha)}{\sqrt{2\pi}E'}\sqrt{c-x_1}, \tag{8}$$

where the superscripts + and − refer, respectively, to the upper and lower surfaces of crack. The energy release rate G for crack initiation can be written as

$$G = \lim_{c\to 0}\frac{1}{2c}\int_0^c \Delta u_2\sigma_{22}dx_1. \tag{9}$$

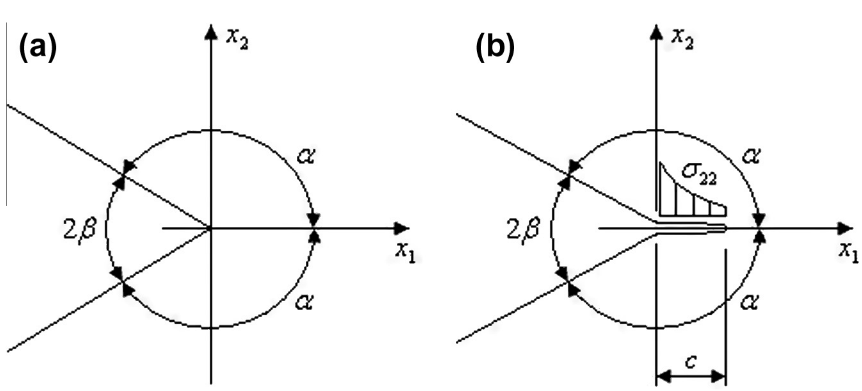

Fig. 1. (a) Configuration of a sharp V-notch with opening angle $2\beta = 2(\pi-\alpha)$;(b) An initial crack with length c emanating from the V-notch's tip.