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Thermal fracture analysis of nonhomogeneous piezoelectric materials using an interaction energy integral method

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ABSTRACT

This paper presents a modified interaction energy integral method to analyze the thermal stress intensity factors (TSIFs) and electric displacement intensity factor (EDIF) in nonhomogeneous piezoelectric materials under thermal loading. This modified method is demonstrated to be domain-independent, even when the nonhomogeneous piezoelectric materials contain interfaces with thermo-electro-mechanical properties. As a result, the method is shown to be convenient for determining the TSIFs and EDIF in nonhomogeneous piezoelectric materials with interfaces. Several examples are shown, and they successfully verify the domain-independence of the present interaction energy integral. The study results also show that the mismatch of material properties can significantly influence the TSIFs and EDIF, particularly when the crack tip is close to the interface. Crack angles and temperature boundary conditions are also shown to significantly influence the TSIFs and EDIF.

1. Introduction

Piezoelectric materials, which can be used in sensors, ultrasonic transducers, and piezoelectric motors, have attracted the attention of many researchers (Sosa, 1992; Suo et al., 1992). The piezoelectric effect is identified as the reversible interaction between a mechanical stress and an electric voltage in such a material, i.e., an applied mechanical stress generates a voltage, and an applied voltage changes the shape of the material. In piezoelectric materials, fracture is an important failure mode (Rao and Sunar, 1993). Further understanding of the fracture mechanism in piezoelectrics will contribute to better applications of these materials.

Many researchers have studied the fracture problems in piezoelectric materials under mechanical loading and thermal loading conditions. Sosa (1992) deduced asymptotic expressions for the electromechanical fields in the vicinity of the crack and studied the effects of the electric field on crack arrest and crack skewing in two-dimensional piezoelectric materials. Zhang et al. (2002) and Zhang and Gao (2004) presented the theoretical analyzes and experimental observations of the failure and fracture behavior of piezoelectric materials. Ueda (2007a,b, 2008) obtained the thermal stress intensity factors (TSIFs) and electric displacement intensity factor (EDIF) in a functionally graded piezoelectric strip by solving a series of singular integral equations. Wang and Mai (2002, 2003) and Wang and Noda (2001) studied the fracture problems in piezoelectric materials under steady state thermal loading and thermal shock loading. Kuna (2006) and Kuna and Ricoeur (2008) defined a thermo-electro-mechanical J-integral and studied the thermal crack problems in smart structures. Rao and Kuna (2010) obtained the stress intensity factors (SIFs) and EDIF using interaction integrals in functionally graded piezoelectric materials subjected to thermal loading. However, the authors did not consider a situation in which the piezoelectric materials contain interfaces. Rao (2009) and Rao and Kuna (2008) also studied the fracture problems in functionally graded magneto-electro-elastic materials subjected to mechanical loading using the domain form of interaction integrals. Yu et al. (2012) used interaction integrals to solve the SIFs and EDIF of piezoelectric materials with complex interfaces. The investigators proved that the interaction integral formulation does not involve any derivatives of mechanical and electric properties. However, they did not consider the thermal fracture problems of the piezoelectric materials.

Although many publications present studies of the mechanical and thermal fracture problems of piezoelectric materials, essentially no reports of thermal fracture problems have been given when the integral domain in piezoelectric material contains interfaces. Hence, in this paper, we aim to develop a modified interaction energy integral method that can be used to obtain the fracture parameters in nonhomogeneous piezoelectric materials with interfaces and to study the effects of material discontinuities on the TSIFs and EDIF.

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2. The basic equations for piezoelectric materials under thermal loading

In this section, first, the temperature field for the thermal fracture problem in piezoelectric materials is determined. Then, the governing equations for a piezoelectric media subjected to thermal loading in the absence of body forces, electric charges, and heat sources are presented.

The temperature field is assumed to satisfy the 2-D steady Fourier heat conduction equation,

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} = 0, \tag{1}$$

where k_x and k_y are the thermal conductivities along the x-axis and y-axis. As shown in Fig. 1, the applied boundary conditions are

$$\begin{aligned} &T|_{(0\leqslant y\leqslant L, x=0)} = T_1, \quad T|_{(0\leqslant y\leqslant L, x=W)} = T_2, \\ &\left.\frac{\partial T}{\partial y}\right|_{(0\leqslant x\leqslant W, y=0)} = 0, \quad \left.\frac{\partial T}{\partial y}\right|_{(0\leqslant x\leqslant W, y=L)} = 0. \end{aligned} \tag{2}$$

In the Finite Element Method (FEM), the temperature distribution in an element can be expressed as

$$T^e(x,y) = \mathrm{N}T^e = \sum_{i=1}^{m} N_i T_i, \tag{3}$$

where N is the shape function in which the ith component is N_i and T^e is the node temperature in which the ith component is T_i. In this paper, $m = 4$ denotes that a 4-node element is adopted. The integration of the Fourier heat conduction equation leads to

$$\int_A N_i \left[k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} \right] dxdy = 0. \tag{4}$$

The equation for the temperature of the element can be written as

$$[\mathbf{K^e}]\{\mathbf{T^e}\} = 0, \tag{5}$$

where the heat conductance matrix of the element, given as $\mathbf{K^e}$, is

$$[\mathbf{K^e}] = \int_A \left[\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + \frac{\partial N^T}{\partial y}\frac{\partial N}{\partial y} \right] dxdy. \tag{6}$$

The assembly of the temperature equations of every element leads to the global temperature equations group

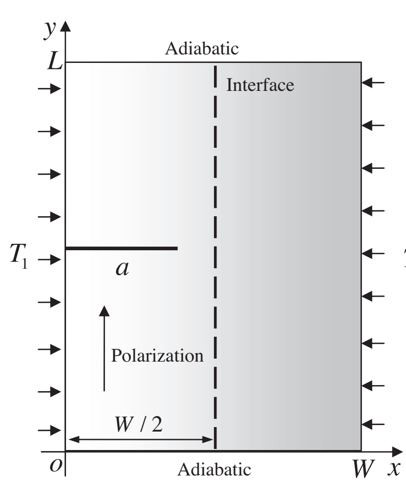

Fig. 1. An edge crack in a nonhomogeneous piezoelectric plate containing an interface.

$$[\mathrm{K}]\{\mathrm{T}\} = \{f\}, \tag{7}$$

where [K] is the global stiffness matrix, {T} is the vector of nodal temperature, and $\{f\}$ is the vector of the corresponding element force. Using Eq. (7) and the temperature boundary conditions, the temperature field can be calculated.

The governing equations for a piezoelectric medium subjected to thermal loading in the absence of body forces, electric charges, and heat sources are given below.

The constitutive equations for the piezoelectric material are

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - e_{lij}E_l - \lambda_{ij}\Delta T \\ D_i &= e_{ikl}\varepsilon_{kl} + \kappa_{il}E_l - \chi_i\Delta T \end{aligned} \quad (i,j,k,l = 1,2,3); \tag{8}$$

the kinematic equations are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j1} + u_{j,i1}), \quad E_i = -\phi_{,i}; \tag{9}$$

and the equilibrium equations are

$$\sigma_{ij,j} = 0, \quad bD_{i,i} = 0, \quad h_{i,i} = 0, \tag{10}$$

where $h_i = -k_{ij}\Delta T$, h_i and k_{ij} are the heat flux and the coefficients of heat conduction, respectively. Additionally, ΔT is the difference of the absolute temperature between the temperature and the stress-free reference temperature T_0. In Eqs. (8)–(10), a comma denotes partial differentiation, and the repeated indices denote summation; u_i, σ_{ij}, ϕ, D_i, and E_i are the elastic displacements, stresses, the electric potential, electric displacements, and the electric field, respectively; C_{ijkl}, e_{lij}, κ_{il} are the elastic stiffness, piezoelectric constants, and dielectric permittivity, respectively.

In Eq. (8), the temperature stress coefficients λ_{ij} and the pyroelectric displacement constants χ_i, which are related to the tensors of the thermal expansion coefficients α_{kl} and the pyroelectric field constants g_l, are shown below:

$$\begin{aligned} \lambda_{ij} &= C_{ijkl}\alpha_{kl} - e_{lij}g_l, \\ \chi_i &= e_{ikl}\alpha_{kl} + \kappa_{il}g_l. \end{aligned} \tag{11}$$

We define ε^m_{kl} and E^m_l, which are the electromechanical parts of the total strain ε^t_{kl} and the total electric field E^t_l, as

$$\begin{aligned} \varepsilon^m_{kl} &= \varepsilon^t_{kl} - \varepsilon^{th}_{kl} = \varepsilon^t_{kl} - \alpha_{kl}\Delta T, \\ E^m_l &= E^t_l + E^{th}_l = E^t_l + g_l\Delta T, \end{aligned} \tag{12}$$

where ε^{th}_{kl} and E^{th}_l are the thermo-electro-mechanical components of strain and the electric field, respectively.

According to Eq. (12), the constitutive equations can be simplified as

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon^m_{kl} - e_{lij}E^m_l, \\ D_i &= e_{ikl}\varepsilon^m_{kl} + \kappa_{il}E^m_l. \end{aligned} \tag{13}$$

According to Hwu (2008), Eq. (13) is equivalent to the following set of equations:

$$\begin{aligned} \varepsilon^m_{ij} &= S_{ijkl}\sigma_{kl} + \eta_{kij}D_k, \\ E^m_i &= -\eta_{ikl}\sigma_{kl} + \beta_{ik}D_k. \end{aligned} \tag{14}$$

Using the relationship between the indices $11 \to 1$, $22 \to 2$, $33 \to 3$, $23 \to 4$, $31 \to 5$, $12 \to 6$, Eq. (13) can be written in Voigt notation as

$$\begin{aligned} \sigma_\alpha &= C_{\alpha\beta}\varepsilon^m_\beta - e_{s\alpha}E^m_s, \\ D_i &= e_{i\beta}\varepsilon^m_\beta + \kappa_{is}E^m_s, \end{aligned} \tag{15}$$

where $\alpha, \beta = 1, \ldots, 6$ and $i, s = 1, 2, 3$.

For convenience, we define a generalized matrix C_{ek} for the plane strain state as