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Size-dependent effective elastic moduli of particulate composites with interfacial displacement and traction discontinuities

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ABSTRACT

This work aims at estimating the size-dependent effective elastic moduli of particulate composites in which both the interfacial displacement and traction discontinuities occur. To this end, the interfacial discontinuity relations derived from the replacement of a thin uniform interphase layer between two dissimilar materials by an imperfect interface are reformulated so as to considerably simplify the characteristic expressions of a general elastic imperfect model which is adopted in the present work and include the widely used Gurtin–Murdoch and spring-layer interface models as particular cases. The elastic fields in an infinite body made of a matrix containing an imperfectly bonded spherical particle and subjected to arbitrary remote uniform strain boundary conditions are then provided in an exact, coordinate-free and compact way. With the aid of these results, the elastic properties of a perfectly bonded spherical particle energetically equivalent to an imperfectly bonded one in an infinite matrix are determined. The estimates for the effective bulk and shear moduli of isotropic particulate composites are finally obtained by using the generalized self-consistent scheme and discussed through numerical examples.

1. Introduction

In the mechanics of heterogeneous or composite materials, the interface between two constituent phases is said to be perfect if and only if both the displacement and traction vectors are continuous across it. Otherwise, it is qualified as being imperfect. An imperfect interface is further referred to as being linear or nonlinear according as the relations describing its displacement and traction discontinuities are linear or nonlinear. The situations in which linear or nonlinear imperfect interfaces occur and have to be taken into account are numerous in the mechanics of composites. For example, two constituent phases may be not firmly bonded together or the interface between two dissimilar phases may exhibit non negligible surface energy excess. The present work aims at estimating the size-dependent effective elastic moduli of particulate composites in which both the interfacial displacement and traction discontinuities are present and characterized by linear elastic relations.

Although a great number of works have been dedicated to accounting for the effects of linear imperfect interfaces on the effective elastic moduli of composites (see, e.g., Benveniste, 1985; Brisard et al., 2010; Chen and Dvorak, 2006; Chen et al., 2007; Duan et al., 2005a,b, 2007a,b, 2009; Hashin, 1990, 1991, 1992; Javili et al., 2013; Kushch et al., 2011; Quang and He, 2007, 2008, 2009; Sharma and Ganti, 2004), the interfacial models which have been used are almost exclusively limited to the spring-layer model and the Gurtin–Murdoch model. In the former, the traction vector is continuous across an interface while the displacement vector presents an interfacial jump linearly related to the traction vector. In the latter, the displacement vector is continuous across an interface while the traction vector suffers an interfacial jump which must satisfy the Young–Laplace equation where the surface stress tensor intervenes and is related linearly to the surface strain tensor. However, it is known (see, e.g., Hashin, 2002, 2006, 2008) that the spring-layer and Gurtin–Murdoch interface models are included in a general elastic imperfect interface model. Precisely, by making an asymptotic analysis for an interphase of small uniform thickness between two phases with the purpose of replacing the interphase by an imperfect interface of null thickness, the displacement and traction jump relations governing the imperfect interface can be deduced and characterize a general elastic imperfect interface model. According as the interphase is much softer or stiffer than the neighboring phases, the general interface model reduces to the spring-layer or Gurtin–Murdoch interface model.

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Thus, the general interface model has the advantage of covering not only the spring-layer and Gurtin–Murdoch interface models as two particular extreme cases but also the intermediate cases between them.

In the present work, the general elastic imperfect interface model described above is adopted in order to account for the effects of imperfect interfaces on the effective elastic moduli of particulate composites. The interfacial displacement and traction jump relations characterizing that model are reformulated in such a way that they take simpler compact but equivalent forms particularly convenient for later use. The important problem of determining the elastic fields in an infinite body made of a matrix containing an imperfectly bonded spherical particle and subjected to arbitrary remote uniform strain boundary conditions is then solved in an analytically exact and coordinate-free way. The results obtained for this problem make it possible to directly apply any appropriate micromechanical scheme for estimating the size-dependent effective elastic moduli of particulate composites. In the present work, the method proposed by Duan et al. (2007a), which consists in replacing an imperfectly bonded spherical particle by a perfectly bonded equivalent one in an infinite matrix, is used together with the generalized self-consistent scheme.

Owing to the generality and versatility of the interface model adopted, the elastic fields obtained in the present work for an infinite matrix with an imperfectly bonded spherical particle are new and allows us to retrieve the relevant elastic fields reported in the literature when the spring-layer or Gurtin–Murdoch is employed (see, e.g., Hashin, 1991; Zhong and Meguid, 1997; Sharma and Ganti, 2004; Duan et al., 2007a). In addition, the expressions for the elastic fields are given in a coordinate-free way and hold for any remote uniform strain boundary conditions. The results presented for the effective bulk and shear moduli of a composite with imperfectly bonded spherical particles are also new and include as particular cases the corresponding results given in the literature when the spring-layer or Gurtin–Murdoch is used. Thus, the present work unifies and extends: (i) the results in the literature for the elastic fields in an infinite elastic isotropic body with an imperfectly bonded elastic isotropic spherical particle; (ii) those in the literature for the size-dependent elastic effective bulk and shear moduli of isotropic particulate composites with imperfectly bonded spherical particles.

The rest of the paper is structured as follows. In the next section, the physical background and general expressions of the general elastic imperfect interface model are recalled in the totally anisotropic case. The interfacial displacement and traction relations are reformulated so as to take compact and simple forms. They are particularized to the isotropic case and shown to include as particular cases the spring-layer and Gurtin–Murdoch interface models. In Section 3, the elastic displacement, strain and stress fields in an infinite isotropic matrix containing an imperfectly bonded isotropic spherical particle are derived first for a remote uniform isotropic strain boundary condition, then for a remote uniform shear strain boundary condition and finally for any remote uniform strain boundary condition. It is also shown how to find the corresponding particular results when the spring-layer or Gurtin–Murdoch interface model is used. In section 4, the replacement procedure of Duan et al. (2007a) and the corresponding energy equivalency condition are first recalled. The elastic properties of an equivalent spherical particle are then deduced in the case where the general imperfect interface model is used. The effective bulk and shear moduli of an isotropic composite with imperfectly bonded spherical particles are finally estimated by the generalized self-consistent method. In section 5, numerical examples are given to illustrate some results and make discussions on them. In Section 6, a few concluding remarks are drawn.

2. Interface models

The composite under consideration consists of a matrix in which particulate inhomogeneities are embedded via imperfect interfaces. Let Ω be the 3D domain occupied by a representative volume element (RVE) of the composite. The boundary of Ω is symbolized by $\partial\Omega$. The subdomains of Ω inhabited by a typical inhomogeneity and the matrix are denoted by $\Omega^{(1)}$ and $\Omega^{(2)}$, respectively. The interface between $\Omega^{(1)}$ and $\Omega^{(2)}$ is designed by Γ with the unit normal vector $\boldsymbol{n}$ oriented from $\Omega^{(1)}$ to $\Omega^{(2)}$. The materials forming $\Omega^{(1)}$ and $\Omega^{(2)}$ are assumed to be individually homogeneous and linearly elastic. Their constitutive behaviour is characterized by Hooke's law

$$\boldsymbol{\sigma} = \mathbb{L}\boldsymbol{\varepsilon} \quad \text{or} \quad \boldsymbol{\varepsilon} = \mathbb{M}\boldsymbol{\sigma}, \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ stand for the Cauchy stress and infinitesimal strain tensors; $\mathbb{L}$ and $\mathbb{M}$ are the fourth-order elastic stiffness and compliance tensors. As usual, $\mathbb{L}$ and $\mathbb{M}$ have the minor and major symmetries and are positive definite. The infinitesimal strain tensor $\boldsymbol{\varepsilon}$ is related to the displacement vector $\boldsymbol{u}$ by

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right]. \tag{2}$$

The general elastic interface model initially proposed by Bövik (1994) and Hashin (2002) and then extended by Benveniste (2006) and Gu and He (2011) will be used to describe the interface Γ between $\Omega^{(1)}$ and $\Omega^{(2)}$. Now, we recall the physical background of this model and reformulate it in an equivalent but more convenient form.

2.1. General anisotropic interface model

The interface model in question is based on a physically meaningful three-phase configuration (Fig. 1(a)) where an interphases $\Omega^{(0)}$ of small uniform thickness h is located between a matrix $\hat{\Omega}^{(2)}$ and a particulate inhomogeneity $\hat{\Omega}^{(1)}$. The interface S_1 between $\hat{\Omega}^{(1)}$ and $\Omega^{(0)}$, and the one S_2 between $\hat{\Omega}^{(2)}$ and $\Omega^{(0)}$, are both assumed to be perfect. In the two-phase one (Fig. 1(b)) the interphase is eliminated and replaced by a zero-thickness imperfect interface located at the middle surface Γ of the interphase, while the neighboring phases $\hat{\Omega}^{(1)}$ and $\hat{\Omega}^{(2)}$ are extended to Γ so as to become $\Omega^{(1)}$ and $\Omega^{(2)}$, respectively. Requiring that the jumps of the displacement vector $\boldsymbol{u}$ and the traction vector $\boldsymbol{t}$ across the interphase $\Omega^{(0)}$ in the three-phase configuration (Fig. 1(a)) be, to within an error of order $0\left(h^2\right)$, equal to the corresponding ones across the zone bounded by the surfaces S_1 and S_2 in the two-phase one (Fig. 1(b)), the jump conditions that the imperfect interface Γ in the two-phase configuration must satisfy can be derived. These interfacial jump conditions characterize the general elastic interface model to be used in the present work.

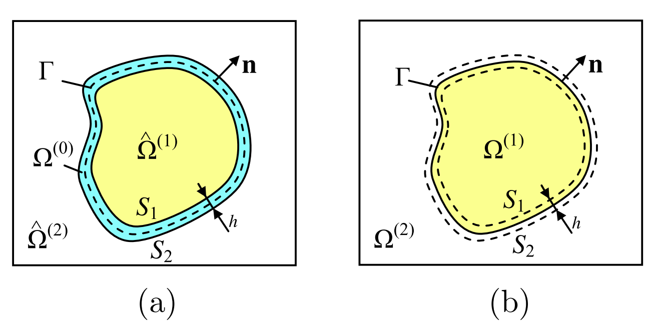

Fig. 1. Replacement of an interphase by an imperfect interface: (a) matrix/interphase/particle configuration; (b) matrix/imperfect interface/particle configuration.