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On micromechanical-statistical modeling of microscopically damaged interfaces under antiplane deformations

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ABSTRACT

A microscopically damaged interface between two elastic half-spaces under anti-plane deformations is modeled using randomly distributed interfacial micro-cracks. The micro-crack length is a continuous random variable following a given probability distribution. The micromechanical-statistical model of the interface, formulated and solved in terms of hypersingular integral equations, is used to estimate the effective stiffness of the interface. The number of micro-cracks per period length of the interface required to homogenize the effective interface stiffness is examined. Also investigated are the effects of the micro-crack length and the crack-tip gap between two neighboring micro-cracks on the effective stiffness.

1. Introduction

Micro-roughness of surfaces (Hamidi et al., 2004) or thermally induced residual stresses during manufacturing processes (Nix, 1989) may give rise to microscopic voids and defects in the interface between two solids which are otherwise perfectly bonded. As illustrated in Fig. 1, for macro-scale analyses, a microscopically damaged interface between two solids may be modeled as a continuous distribution of springs characterized by stiffness parameters.

One of the earlier works dealing with imperfect spring-like interfaces is Jones and Whittier (1967). In Jones and Whittier (1967), the interaction of elastic waves with flexibly bonded interfaces is studied. Since then, many boundary value problems involving spring-like models of imperfect interfaces have been solved (see, for example, Ang, 2007; Fan and Wang, 2003; Margetan et al., 1988; Zhong and Meguid, 1997). Nevertheless, the micromechanical analysis of microscopically damaged interfaces, which includes estimating the effective properties of interfaces, has been investigated by relatively fewer researchers.

Micromechanical models based on continuum mechanics, such as the Voigt approximation, the Reuss approximation, the self-consistent scheme and the three-phase model, for estimating the effective material properties of microscopically heterogenous solids may be found in the research literature (Aboudi, 1991; Christensen, 1990; Li and Wang, 2008). Those models do not attempt to capture all the minute details of the microstructures in the heterogeneous solids. For a more realistic micro-mechanical analysis but one still based on continuum mechanics, the microstructures may be modeled as, for example, randomly generated holes or inclusions in the solids (see Elvin, 1996; Roberts and Garboczi, 1999; Torquato, 2002). Such an approach has been extended by Wang et al. (2012) to the micromechanical analysis of a microscopically damaged interface between two elastic half-spaces under antiplane deformations.

In Wang et al. (2012), the microscopically damaged plane interface is modeled using periodically distributed interfacial micro-cracks. A period length of the damaged interface contains an arbitrary number of randomly positioned micro-cracks. The length of a micro-crack is taken to be a continuous random variable following a given probability distribution. The procedure for estimating the effective stiffness of the interface, which requires solving numerically hypersingular integral equations for the micro-cracks, is described in detail in Wang et al. (2012). The hypersingular integral formulation is advantageous in the micromechanical analysis of the interface (Ang, 2013) as the jump in the displacement across opposite faces of each of the micro-cracks appears directly as an unknown function in the integral equations. Thus, no post-processing of the numerical solution of the integral equations is required to compute the interfacial displacement jump.

Nevertheless, only very limited statistical results for the effective stiffness of the interface are obtained and presented in Wang et al. (2012) using micro-cracks with normally distributed lengths. In reality, the length of a micro-crack may not vary according to a normal distribution. In the present paper, a more realistic

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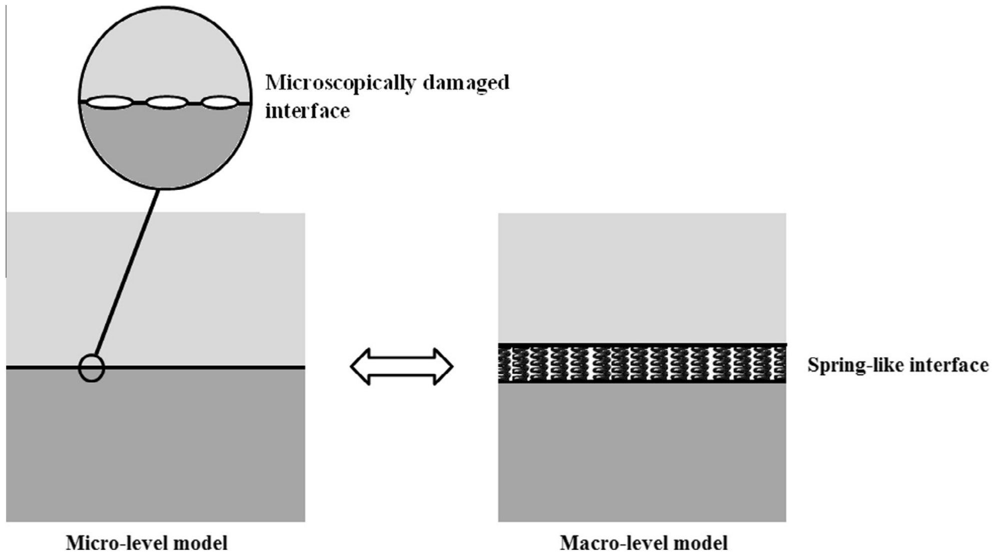

Fig. 1. Micro-level and macro-level models of the damaged interface.

statistical variation of the micro-crack length, based on the chi-squared (χ^2) distribution, is used to generate randomly the length of each micro-crack. The number of micro-cracks per period length of the interface required to homogenize the effective stiffness is examined. Also investigated are the effects of the micro-crack length and the crack-tip gap between two neighboring micro-cracks on the effective stiffness.

2. Micromechanical model

With reference to a Cartesian coordinate frame denoted by $Ox_1x_2x_3$, consider two dissimilar homogeneous anisotropic elastic half-spaces occupying the regions $x_2 > 0$ and $x_2 < 0$. The plane interface $x_2 = 0$ joining the half-spaces is microscopically damaged containing microscopic voids and defects.

The bimaterial undergoes an antiplane elastostatic deformation such that the only non-zero component of the displacement field is along x_3 direction. The elastic displacement $u_3(x_1, x_2)$ and stress $\sigma_{3i}(x_1, x_2)$ along the microscopic portion $0 < x_1 < l$ of the damaged interface may be homogenized by the averaging procedure

$$\begin{aligned} \overline{u}_3(\overline{x}_1, 0^{\pm}) &= \frac{1}{2l}\int_{\overline{x}_1 - l}^{\overline{x}_1 + l} u_3(x_1, 0^{\pm})dx_1, \\ \overline{\sigma}_{3i}(\overline{x}_1, 0^{\pm}) &= \frac{1}{2l}\int_{\overline{x}_1 - l}^{\overline{x}_1 + l} \sigma_{3i}(x_1, 0^{\pm})dx_1, \end{aligned} \tag{1}$$

where $\overline{x}_1$ denotes the midpoint of the microscopic portion of the interface.

In terms of the homogenized field variables $\overline{u}_3$ and $\overline{\sigma}_{3i}$, the macro-level spring model for the interface (see, for example, Hashin, 1991) is defined by

$$\overline{k}(\overline{u}_3(\overline{x}_1, 0^+) - \overline{u}_3(\overline{x}_1, 0^-)) = \overline{\sigma}_{32}(\overline{x}_1, 0^+) = \overline{\sigma}_{32}(\overline{x}_1, 0^-), \tag{2}$$

where $\overline{k}$ is the effective stiffness of the interface. Note that $\overline{u}_3(\overline{x}_1, 0^+) - \overline{u}_3(\overline{x}_1, 0^-)$ gives the homogenized displacement jump across the damaged interface.

The conditions in (2) are also given in Benveniste and Miloh (2001). In Benveniste and Miloh (2001), they are derived using an asymptotic analysis on the elastic fields in an infinitesimally thin layer of an extremely soft material bonded between the elastic half-spaces.

To estimate the effective stiffness $\overline{k}$ in the macro-model defined by (2), Wang et al. (2012) simulated the microscopically damaged interface in Fig. 1 by proposing a micromechanical model in which the microscopic voids and defects of the interface are replaced by periodically distributed interfacial micro-cracks. More specifically, the part of the interface defined by $0 < x_1 < L, x_2 = 0$, contains M interfacial micro-cracks with the tips of the mth crack given by $(a^{(m)}, 0)$ and $(b^{(m)}, 0)$, where

$$0 < a^{(1)} < b^{(1)} < a^{(2)} < b^{(2)} < \cdots < a^{(M)} < b^{(M)} < L.$$

The micro-cracks on the remaining part of the interface lie in the regions where $a^{(m)} + nL < x_1 < b^{(m)} + nL$ for $m = 1, 2, \ldots, M$ and $n = \pm 1, \pm 2, \ldots$. The elastic half-spaces are perfectly bonded on the uncracked parts of the interface. The periodically distributed micro-cracks are traction-free under the action of the antiplane constant shear load given by $\sigma_{3i} = \sigma_{3i}^{(\mathrm{int})}$ at infinity, where $\sigma_{3i}^{(\mathrm{int})}$ is the antiplane shear stress in the bimaterial for the corresponding case where there is no micro-crack on the interface. For the studies here, $\sigma_{3i}^{(\mathrm{int})}$ is chosen such that $\sigma_{32}^{(\mathrm{int})} = S_0$ on all the micro-cracks, where S_0 is a positive constant. A sketch of the micromechanical model is given in Fig. 2.

As derived in Wang et al. (2012), the hypersingular integral equations for the micromechanics model of the microscopically damaged interface are given by

$$\begin{aligned} &\sum_{m=1}^{M} \fint_{a^{(m)}}^{b^{(m)}} \Delta u_3(x_1)\left[\frac{1}{x_1 - \xi_1} + \frac{1}{(L + x_1 - \xi_1)^2} + \frac{1}{(L + \xi_1 - x_1)^2}\right. \\ &\left. + \frac{1}{L^2}\psi^*\left(\frac{L + x_1 - \xi_1}{L}\right) + \frac{1}{L^2}\psi^*\left(\frac{L + \xi_1 - x_1}{L}\right)\right]dx_1 \\ &= -\frac{\pi(\beta^{(1)} + \beta^{(2)})}{\beta^{(1)}\beta^{(2)}}S_0 \quad \text{for } a^{(n)} < \xi_1 < b^{(n)} \quad (n = 1, 2, \ldots, M). \end{aligned} \tag{3}$$

where $\fint$ denotes that the integral is to be interpreted in the Hadamard finite-part sense, $\Delta u_3(x_1) = u_3(x_1, 0^+) - u_3(x_1, 0^-)$ denotes the displacement jump across the opposite faces of the micro-cracks, $\psi^*(x) = \psi_1(x) - 1/x^2, \psi_1(x)$ is the trigamma function, $\beta^{(p)} = \sqrt{C_{44}^{(p)}C_{55}^{(p)} - (C_{45}^{(p)})^2}$ and $C_{44}^{(p)}, C_{45}^{(p)}$ and $C_{55}^{(p)}$ are the elastic moduli of the anisotropic materials in the half-spaces ($p = 1$ for the material in $x_2 > 0$ and $p = 2$ for the material in $x_2 < 0$).

A numerical method for solving (3) for the displacement jump $\Delta u_3(x_1)$ over each of the micro-cracks is described in