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A computational homogenization approach for the yield design of periodic thin plates. Part II : Upper bound yield design calculation of the homogenized structure

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A B S T R A C T

In the first part of this work (Bleyer and de Buhan, 2014), the determination of the macroscopic strength criterion of periodic thin plates has been addressed by means of the yield design homogenization theory and its associated numerical procedures. The present paper aims at using such numerically computed homogenized strength criteria in order to evaluate limit load estimates of global plate structures. The yield line method being a common kinematic approach for the yield design of plates, which enables to obtain upper bound estimates quite efficiently, it is first shown that its extension to the case of complex strength criteria as those calculated from the homogenization method, necessitates the computation of a function depending on one single parameter. A simple analytical example on a reinforced rectangular plate illustrates the simplicity of the method. The case of numerical yield line method being also rapidly mentioned, a more refined finite element-based upper bound approach is also proposed, taking dissipation through curvature as well as angular jumps into account. In this case, an approximation procedure is proposed to treat the curvature term, based upon an algorithm approximating the original macroscopic strength criterion by a convex hull of ellipsoids. Numerical examples are presented to assess the efficiency of the different methods.

1. Introduction

In this joint work, the yield design of periodic thin plate structures is investigated. The first part of this work has been dedicated to the determination of the homogenized strength properties of different periodic plates through the numerical computation of a macroscopic strength criterion by means of finite elements and mathematical programming.

Homogenization theory in the framework of yield design (or limit analysis) of periodic structures has first been proposed in the work of Suquet (1985) and de Buhan (1986), where a proper definition of the macroscopic strength criterion involving the resolution of an auxiliary yield design problem formulated on the unit cell has been given. An analytical determination of the macroscopic strength criterion is very rare (e.g., the case of the multilayered soil under plane strain (de Buhan, 1986)) and often restricted to symmetric unit cell geometries and simple macroscopic loading (Maghous, 1991). Therefore, numerical methods are required, notably to conveniently capture the anisotropy of the homogenized material when preferential reinforcing directions are involved. The numerical resolution of the auxiliary problem can be tackled using incremental elasto-plastic approaches (Marigo et al., 1987) but a more natural method is to perform numerical limit analysis computations directly. This method, in conjunction with a finite element discretization, has been widely applied to different type of structures like porous media (Pastor and Turgeman, 1983; Turgeman and Pastor, 1987), periodic plates solicited in their own plane (Maghous, 1991; Francescato and Pastor, 1998), masonry walls (Sab, 2003), stone columns reinforced soils (Hassen et al., 2013) whereas the first part of this work (Bleyer and de Buhan, 2014) deals with thin periodic plates in bending.

Different numerical techniques have also been used to solve the corresponding optimization problem. In particular, linear programming (LP) associated to a piecewise linearization of the original local strength criterion has been very attractive due to the efficiency of interior point algorithms to solve LP problems. The extension of these algorithms to a wider class of convex programming problems, namely second-order cone programming (SOCP) enables today to solve limit analysis problems with their original nonlinear criterion very efficiently. SOCP has been notably used in the first part of this work to solve the static as well as the kinematic approach of the auxiliary problem.

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Although an important amount of work has been dedicated to the numerical determination of macroscopic strength criteria of heterogeneous media, only a few papers, to the authors' knowledge, have been aimed at estimating the ultimate load of global structures made of such macroscopic strength criteria derived from the homogenization procedure. One can mention the work of Milani et al. (2006) concerning brick masonry or the following papers (Maghous et al., 1998; Hassen et al., 2013) on geotechnical problems. The small amount of work dedicated to this specific aspect is certainly due to the absence of closed-form expressions for the homogenized yield surfaces, which restrains their use to simple failure mechanisms for analytical applications. As regards numerical approaches, the homogenized yield surface has to be approximated, usually by piecewise linearization, to be dealt with. However, the number of hyperplanes describing the surface with a sufficient accuracy can be quite important, which is not desirable for efficient computations.

As regards the specific case of thin plates in bending, which is the scope of the present work, the strength criterion depends on the bending moment only. The yield line method, originally proposed by Johansen (1962), is an efficient upper bound kinematic method which considers only rigid mechanisms separated by yield lines where angular rotation discontinuities occur. Analytical upper bound estimates are, therefore, easily available and a numerical implementation using linear triangular finite element is also possible (Munro and Da Fonseca, 1978), although there are some inherent difficulties due to mesh dependency (Johnson, 1994; Jennings, 1996). However, to obtain tight upper bound estimates of plates in bending, dissipation through curvature has also to be taken into account. This requires to use a quadratic interpolation, at least, of the plate velocity field. Some authors proposed to use C^1-continuous high order finite elements to perform the upper bound approach (Capsoni and Corradi, 1999; Le et al., 2010) but better estimates have been obtained by the present authors using only C^0- continuous element (Bleyer and de Buhan, 2013a), dissipation being produced by curvature as well as angular rotation discontinuities. The aim of this work is, thus, to perform yield design computations of plate structures by adapting these numerical methods to the case of complex anisotropic strength criteria computed from homogenization. It will be shown that the yield line method can easily be extended to these criteria without much difficulties, whereas a specific approximation procedure will be required for a complete finite element upper bound approach. It should be noticed that the proposed methods aim at taking advantage of the efficiency of SOCP solvers, which enable to manipulate nonlinear strength criteria, so that a more efficient approximation procedure than piecewise linearization is possible.

Section 2 will first be devoted to the extension of the yield line method to complex strength criteria and an analytical example on a simply supported rectangular plate made of a reinforced material will be presented. Section 3 will present a procedure to approximate a numerically computed three-dimensional yield surface by a convex hull of ellipsoids so that a finite element kinematic approach can be formulated and treated by SOCP solvers. Finally, numerical examples making use of some macroscopic strength criteria previously computed in Part I (Bleyer and de Buhan, 2014) will be investigated.

2. A first attempt at evaluating the bearing capacity of heterogeneous thin plates by the yield line method

2.1. Yield line method for a numerically computed strength criterion

The yield line method is a simple upper bound approach for the yield design of plates in bending which considers only rigid mechanisms separated by yield lines, where jumps of angular velocity have to be taken into account in the expression of the maximum resisting work. The formulation of the corresponding upper bound yield design problem reads as:

$$\underline{Q} \in \Lambda \Rightarrow \forall \widehat{u}\text{K.A. with } \underline{q}, \quad P_{ext}(\widehat{u}) \leqslant P_{rm}(\widehat{u})$$

where $P_{ext}(\widehat{u})$ is the work of external loads in the kinematically admissible velocity field $\widehat{u}$ and

$$P_{rm}(\widehat{u}) = \int_{\Gamma} \Pi([\![\theta_n]\!];\underline{n})dl$$

is the maximum resisting work associated with a set of yield lines Γ of unit normal $\underline{n}$ and angular velocity jumps $[\![\theta_n]\!]$ across Γ following the normal $\underline{n}$. Hence, $\Pi([\![\theta_n]\!];\underline{n})$, which corresponds to a particular value of the support function of the strength criterion associated with rotation discontinuities, is defined as:

$$\Pi([\![\theta_n]\!];\underline{n}) = \sup_{\underline{\underline{M}}\in G} M_{nn}[\![\theta_n]\!] = \Pi(\underline{\underline{\chi}} = [\![\theta_n]\!]\underline{n}\otimes\underline{n})$$

We now consider that the local strength criterion of the plate is a macroscopic strength criterion G^{hom} obtained from a homogenization procedure, as described in the first part of this work. In particular, it is described by its support function $\Pi_{hom}(\underline{\underline{\chi}})$. Using the previous remarks, we have:

$$\Pi([\![\theta_n]\!];\underline{n}) = \Pi_{hom}([\![\theta_n]\!]\underline{n}\otimes\underline{n}) = |[\![\theta_n]\!]|\Pi_{hom}(\underline{n}\otimes\underline{n})$$

since support functions are positively 1-homogeneous. Let $(\underline{e}_x, \underline{e}_y)$ be the orthonormal frame attached to the periodic unit cell, then,

$$\Pi([\![\theta_n]\!];\underline{n}) = |[\![\theta_n]\!]|\Pi_{hom}(\cos^2\alpha(\underline{e}_x\otimes\underline{e}_x) + \sin^2\alpha(\underline{e}_y\otimes\underline{e}_y) + \sin 2\alpha(\underline{e}_x\overset{s}{\otimes}\underline{e}_y)) = |[\![\theta_n]\!]|\Pi_0(\alpha)$$

where α is such that $\underline{n} = \cos\alpha\underline{e}_x + \sin\alpha\underline{e}_y$. Therefore, the support function of rotation discontinuities is entirely described by function $\Pi_0(\alpha)$ depending on the sole normal orientation angle α. This function can be determined by solving a series of auxiliary yield design problems attached to the unit cell with a macroscopic curvature of the form: $\chi_{xx} = \cos^2\alpha, \chi_{yy} = \sin^2\alpha$ and $\chi_{xy} = \sin 2\alpha/2$ for different values of α. Fig. 1 represents such a function $\Pi_0(\alpha)$ corresponding to the reinforced plate example presented in the first part of this work is represented. The dependance of this function with respect to α is characteristic of the reinforced plate anisotropy.

Finally, once the geometry of the yield line mechanism has been fixed, $\Pi_0(\alpha)$ can be computed for all yield line normal orientations

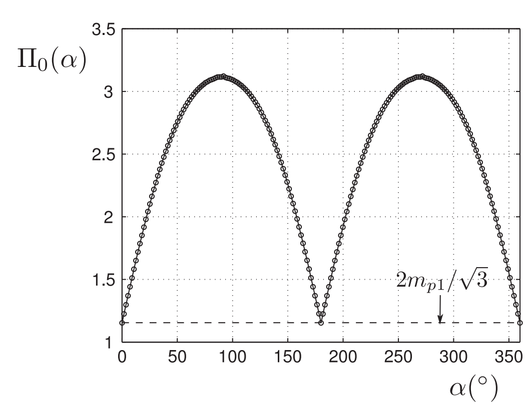

Fig. 1. Function $\Pi_0(\alpha)$ for the reinforced plate example (Bleyer and de Buhan, 2014). The circles correspond to numerically computed values obtained from the resolution of auxiliary yield design problems on the unit cell. The horizontal dotted line corresponds to the value of $\Pi_0(\alpha)$ in the case of an unreinforced von Mises plate with ultimate bending moment $m_{p1} = 1$.