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Influence of transverse curvature on the stability of pre-stressed helical structures

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ABSTRACT

This study is concerned with the stability characteristics of helix shaped structures made of anisotropic, pre-stressed, thin flanges arranged in such a way as to enable and develop multi-stability. Previous research on similar structures assumed the structural response of the flanges to be one-dimensional due to the narrow width of the pre-stressed members in comparison to their length. In this work, a refined two-dimensional model of the flanges is employed to model the influence of transverse curvature as well as the membrane strain energy associated with the non-zero Gaussian curvature deformations. While longitudinal curvature changes and twist are inherent to the geometry of the helices; the transverse curvature results from a consideration of boundary effects and the minimisation of the (expensive) membrane elastic energy. A qualitative study of the changes in transverse curvature reveals ways of simplifying the two-dimensional model into a simpler, closed form, one-dimensional version applicable to helices with relatively narrow flanges. Correlation is found between experimental results, finite element modelling and analytical predictions for the two models.

1. Introduction

Large deformations in conventional engineering design are often associated with failure, frequently resulting in the sudden collapse of the structure, as observed in the buckling of a slender beam in compression. To the contrary, morphing structures are designed to be reconfigured between radically different states while keeping their load carrying capability and structural integrity throughout the deformations. Owing to their variable geometry, low weight and reduced complexity, research efforts are being made to apply morphing structures to new aircraft design. Many concepts depend on continuously actuated structures (Bartley-Cho et al., 2004; Berton, 2006; Wildschek et al., 2009; Daynes and Weaver, 2012a,b). However, if the structure possesses multiple stable states, energy is needed only to change the shape, not to hold it. The geometrical stable configurations correspond to minima in the internal energy state of the structure and can occur owing to several phenomena when using fibre reinforced plastics (FRP). Multi-stability in composite material was first reported by Hyer (1981,1982) and was explained by the combination of residual stresses induced during the cure cycle and geometric nonlinearities in a non-symmetric laminated FRP lay-up. More recent studies investigated the effect of Gaussian curvature (Kebadze et al., 2004; Seffen, 2007; Guest et al., 2011; Brinkmeyer et al., 2012), fibre pre-stress (Daynes et al., 2008, 2010), plastic deformation (Guest and Pellegrino, 2006) or bending stiffness tailoring (Daynes et al., 2011) as a mean to introduce multi-stability.

The background of this research concerns the multi-stable deployable composite device presented by Lachenal et al. (2012). Their study focussed on the stability of helices made of two pre-stressed narrow FRP strips joined by metallic spokes. Potential applications include deployable booms for space structures; however the present concept could find applications as strain energy storage structure, as in (Lachenal et al., 2013) or as a stiffness tailored integrated twist morphing structure device (Lachenal et al., 2014). In their previous research, the combination of pre-stress, material properties and geometry was explored revealing bistable devices with periodically nonlinear, yet tailorable deformation responses; enabling, for example, the helix to be stable in a tightly coiled or fully extended configuration. The analytical model used one-dimensional elements, hence zero transverse curvature was assumed in addition to inextensional deformations of the pre-stressed members, resulting in a compact analytical model. While the corresponding initial experimental results matched closely with both analytical and finite element model (FEM) results, this one-dimensional model was only validated with a laminate free

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of off-axis fibres for the strips constituting the helix. Later experiments with helices presenting off-axis laminates revealed the limitations of the one-dimensional assumption and thus triggered the present study.

In this paper, the model by Lachenal et al. (2012) is enhanced by lifting the assumption of inextensional deformations and zero transverse curvature. As such, the one-dimensional model is complemented by the analytical model from Giomi and Mahadevan (2011) including the effects of transverse curvature within anisotropic strips. A related study was previously presented by Galletly and Guest (2004) on the modelling of bistable, composite, tape-spring like structures called *slit tubes*. Their work showed that slit tubes made of an anti-symmetric lay-up possess a second, stable, coiled equilibrium depending on the width of the structure. It was demonstrated that a non-constant curvature develops across the section of the tube, particularly close to the edges of the section where a "boundary layer" appears. While for large cross sections the boundary layer was small and did not affect the stability of the slit tubes, for small cross sections the effect of this boundary significantly affected the stability of the tube.

Our new work presents two models describing the stability of helical structures: a two-dimensional model accounting for the varying transverse curvature of the strips composing the helices and a simpler, closed form, one-dimensional model with restricted design space. The two dimensional model captures the mechanics driving the development of transverse curvature. It shows in a novel way that two competing effects occur across the width of the strips and that the resulting balance of strain energy is highly dependent on the relative size of the cross section. This study reveals ways of reducing the complexity of the two-dimensional model for the helix back to a simpler, yet more complete one-dimensional model in comparison with our prior work (Lachenal et al., 2012); yielding an elegant relationship between stability, material stiffnesses and structure geometry.

The article is structured as follows. A brief introduction to the concept of pre-stressed strips arranged into helices is given first in §2 followed by the strain energy formulation and the conditions for stability. Details of the elastic deformations occurring throughout the transformation of the structure are presented in §3: the key features of the model from Giomi and Mahadevan are highlighted followed by a qualitative study on the effect of the width on the transverse curvature of the flanges. The study of the influence of the transverse curvature on the stability landscape reveals a simplification of the two-dimensional model and is presented with the associated assumptions. Two cases of helices are detailed in §4 to illustrate the improved analytical model; a third case study exemplifies the simplified model. The finite element model of the morphing structure is described in §5. Results from the analytical model, FEM and experimental work are discussed in §6. Section 7 concludes the paper.

2. Morphing composite helix

2.1. Description

The structure in a twisted configuration is depicted in Fig. 1(a). It consists of at least two flanges (or "strips") of dimensions $L \times W$ kept apart by a set of rigid spokes of height $H = 2R$ where R is defined in§3. The device can twist by an angle ϕ by applying opposite moments about the X-axis at the extremities of the structure; as such, the helix has an infinite number of configurations, from tightly coiled to fully extended (respectively shown in black and light grey in Fig. 1(b)). A global coordinate system (X,Y,Z) is attached to the fixed end of the device while a local coordinate system (x,y,z) is attached to each strip (see Fig. 2). The angle of helix, given between the local x-axis and the global X-axis, defines the configuration of the helix, as in Fig. 1(a). Multi-stability is achieved by imposing a state of pre-stress to the strips: in the present case a distributed bending moment m_x is introduced by manufacturing the parts on a cylindrical mould of radius R_i and then by subsequent flattening, as illustrated in Fig. 2. It is worth noting that this model neglects the local constraints and end-effects imposed by the spokes on the strips, but such effects are of little consequence for present purposes.

2.2. Two-dimensional, extensional, helix model

Contrary to the study in the prior work of Lachenal et al. (2012), the strips constituting the helix are considered as two-dimensional elements of length L and width W. It is assumed in this two-dimensional model that the deformations are extensional; therefore bending and membrane strains are present in the structure, thus the total strain energy can be expressed as (Kollar and Springer, 2003)

$$U = \frac{n}{2}\int_0^L \int_{-W/2}^{W/2} \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \Delta\boldsymbol{\kappa} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \Delta\boldsymbol{\kappa} \end{bmatrix} dxdy \tag{1}$$

where $\Delta\boldsymbol{\kappa}$ is the tensor change of curvature and $\boldsymbol{\varepsilon}^0$ is the mid-plane strain tensor, both expressed in the local (x,y,z) coordinate system shown in Fig. 2. The superscript T denotes the transpose of the tensors. **A**, **B** and **D** are the in-plane, bending-extension coupling and flexural stiffness matrices, respectively as defined in classic lamination theory (CLT) (Jones, 1999). Finally, n is the number of flanges. Developing Eq. (1) yields three terms

$$U = \frac{n}{2}\int_0^L \int_{-W/2}^{W/2} \boldsymbol{\varepsilon}^{0^{\mathrm{T}}}\mathbf{A}\boldsymbol{\varepsilon}^0 dxdy + n\int_0^L \int_{-W/2}^{W/2} \varepsilon^{0^{\mathrm{T}}}\mathbf{B}\Delta\boldsymbol{\kappa} dxdy + \frac{n}{2}\int_0^L \int_{-W/2}^{W/2} \Delta\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{D}\Delta\boldsymbol{\kappa} dxdy \tag{2}$$

Eq. (2) shows three deformation patterns: a stretching deformations model, a bending deformations model plus a bending-extension coupling term.

We first consider the bending deformations, thus the strain energy takes the form (Kollar and Springer, 2003)

$$U_b = \frac{n}{2}\int\int \Delta\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{D}\Delta\boldsymbol{\kappa} dxdy \tag{3}$$

As in the previous work of the authors, it is assumed that the changes of x- and xy- curvatures (respectively $\Delta\kappa_x$ and $\Delta\kappa_{xy}$) are constant over L and W. It is demonstrated in §3 that the y-axis change of curvature (transverse curvature) is a non-constant quantity across W thus Eq. (3) becomes

$$U_b = \frac{n}{2}LW(D_{11}\Delta\kappa_x^2 + D_{66}\Delta\kappa_{xy}^2 + 2D_{16}\Delta\kappa_x\Delta\kappa_{xy}) + \frac{n}{2}L\int_{-W/2}^{W/2} D_{22}\Delta\kappa_y^2 + 2D_{12}\Delta\kappa_x\Delta\kappa_y + 2D_{26}\Delta\kappa_y\Delta\kappa_{xy} dy \tag{4}$$

The membrane strain energy resulting from the axial deformation is (Kollar and Springer, 2003)

$$U_s = \frac{n}{2}\int\int \boldsymbol{\varepsilon}^{0^{\mathrm{T}}}\mathbf{A}\boldsymbol{\varepsilon}^0 dxdy \tag{5}$$

It is later assumed, as in Galletly and Guest (2004), Giomi and Mahadevan (2011), that only x-axis strains result from the deformation of the helix; thus Eq. (5) reduces to

$$U_s = \frac{n}{2}LA_{11}\int_{-W/2}^{W/2} \varepsilon_x^{0^2} dy \tag{6}$$

Eq. (2) reveals a bending-extension strain energy term due to the presence of the **B** matrix in the laminate properties. As detailed above, it is assumed that only x-direction strains arise from the