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In-plane elastic wave propagation and band-gaps in layered functionally graded phononic crystals

S.I. Fomenko [a], M.V. Golub [a,*], Ch. Zhang [b], T.Q. Bui [b], Y.-S. Wang [c]

[a] Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar 350040, Russia
[b] Chair of Structural Mechanics, Department of Civil Engineering, University of Siegen, Paul-Bonatz Strasse 9-11, D-57076 Siegen, Germany
[c] Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, PR China

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ABSTRACT

In-plane wave propagation in layered phononic crystals composed of functionally graded interlayers arisen from the solid diffusion of homogeneous isotropic materials of the crystal is considered. Wave transmission and band-gaps due to the material gradation and incident wave-field are investigated. A classification of band-gaps in layered phononic crystals is proposed. The classification relies on the analysis of the eigenvalues of the transfer matrix for a unit-cell and the asymptotics derived for the transmission coefficient. Two kinds of band-gaps, where the transmission coefficient decays exponentially with the number of unit-cells are specified. The so-called low transmission pass-bands are introduced in order to identify frequency ranges, in which the transmission is sufficiently low for engineering applications, but it does not tend to zero exponentially as the number of unit-cells tends to infinity. A polyvalent analysis of the geometrical and physical parameters on band-gaps is presented.

1. Introduction

Functionally graded materials (FGMs) are advanced composites consisting of two or more material phases, characterized by a gradual variation in composition and structure in some spatial directions. The concept of FGMs was introduced in 1984 by a group of material scientists in Japan (Woo and Meguid, 2001; Shen, 2009). One of the superior properties of the FGMs compared to the classical composites is that the continuous gradation in material properties can overcome the interfacial problems typical for most layered composite structures. Owing to their great advantages in a variety of engineering applications, FGMs are essentially designed to take advantages of desirable characteristics of each of the constituent phases.

In recent years, there has been a great deal of works on the analysis of propagation of elastic waves in periodic composite structures or phononic crystals. Phononic crystals are functional composite materials composed of periodic arrays of two or more materials with different material properties and mass densities. In general, the periodicity of phononic crystals may be in one-, two-, or three dimensions with different scatterers, respectively. By owning to the great advantages in a broad range of engineering applications, the wave propagation phenomena in composite materials and structures particularly play an important role in design of new devices in such engineering applications. Basically, the most important properties of phononic crystals lie in the mechanical or acoustical waves, which have specific frequency ranges in which they cannot propagate within the periodic structures. Thus, the frequency ranges that are forbidden for wave propagation are usually called phononic band-gaps or stop-bands, and the occurrence of such band-gaps in periodic elastic structures is caused by the multiple wave scattering at the interfaces between different materials (Brillouin, 1946; Maldovan and Thomas, 2009).

The potential of the methods based on elastic waves for damage detection was demonstrated last century (Viktorov, 1967; Achenbach, 1973; Alleyne and Cawley, 1992). Ultrasonic non-destructive methods require precise and accurate wave excitation and signal reception techniques, which can be realized in particular by the introduction of special elements like phononic crystals with good filtering properties into the actuators and sensors. An example is the acoustic sensor system using resonances of two-dimensional (2D) phononic crystals made up of a steel plate having two regular arrays of holes and a cavity in-between (Zubtsov et al., 2012). Since one-dimensional (1D) phononic crystals or multilayered periodic laminates can be fabricated more easier than two-dimensional (2D) and three-dimensional (3D) phononic crystals, they should be

* Corresponding author. Tel.: +7 9628654432.
E-mail addresses: sfom@yandex.ru (S.I. Fomenko), m_golub@inbox.ru (M.V. Golub), c.zhang@uni-siegen.de (Ch. Zhang), bui-quoc@bauwesen.uni-siegen.de (T.Q. Bui), yswang@bjtu.edu.cn (Y.-S. Wang).

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investigated in details due to their novel acoustic properties, which have potential engineering applications (Saini et al., 2007). An experimental investigation of phononic band-gaps for a normal wave incidence in a 1D periodic SiO_2/poly (methyl methacrylate) multilayered film at gigahertz frequencies using Brillouin spectroscopy was performed by Gomopoulos et al. (2010). Though many works have been performed on phononic crystals, design of an optimal phononic crystal remains a complex task and elastic wave propagation in 1D phononic crystals is not fully understood yet. The present study aims at developing efficient and accurate methods for fast calculations of band-gaps, which can be applied in the optimization procedure in order to design 1D FGM phononic crystals with demanded properties, and for reliable investigations of the corresponding wave phenomena.

Many previous efforts have particularly been devoted to the simulation and analysis of wave propagation and scattering problems in FG materials and structures (Liu et al., 1991; Sobczak and Drenchev, 2013; Jha et al., 2013). Besides purely numerical methods, including finite difference time domain method (Berezovski et al., 2003; Vollmann et al., 2006), finite element method (Chakraborty and Gopalakrishnan, 2003; Santare et al., 2003) and hybrid numerical methods (Liu et al., 1991; Han et al., 2001) that are more suitable for finite FGM bodies, there are several semi-analytical approaches based on the solution of boundary-value problems in spatial Fourier transform domains (Babeshko et al., 1987; Liu et al., 1999). The semi-analytical approaches can be classified into two groups for convenience. The first group involves explicit mathematical models of FGMs such as the direct integration of differential equations with variable coefficients (Sato, 1959), including modulating function method for Green's matrix (Babeshko et al., 1987) and Peano expansion method for matricants (Shuvalov et al., 2005). The methods from the second group are based on the approximation of an FG layer by a set of sub-layers in which the displacement vectors have explicit expressions. For example, approximations of continuous elastic moduli within the layer by different functions (step-wise, exponential, linear or special power-law) (Liu et al., 1999; Ke and Wang, 2006; Matsuda and Glorieux, 2007; Ting, 2011) and the propagator technique (Gilbert, 1983; Kutsenko et al., 2013) have been used. The simplest layer model (LM) was proposed by using step-wise approximation with homogeneous elastic sub-layers.

Among the most popular methods for layered medium is the transfer matrix method that dates back to the works of Thomson (1950), Petrashen (1952) and Haskell (1953). The LM in conjunction with the T-matrix method seems to be more convenient for numerical calculations of band-gaps in 1D multilayered FG phononic crystals because of fast implementation. However, the equivalence of the LM and the explicit models of the FG layer is not so evident since the explicit models presuppose the first derivatives of the material properties in the governing equations which is excluded in the LM. For example, the replacement of FGMs by a set of homogeneous sub-layers is poor for static problems of indentation (Aizikovich et al., 2011). On the other hand, a comparative analysis of Green's matrixes and surface waves derived by both LM method and the direct integration method with modulating functions have been implemented for in-plane problem in Glushkov et al. (2012). The study has shown that the LM with sufficient number of sublayers is suitable for the investigation of bulk and surface waves in the layered media. Both of these two approaches have been used also in Golub et al. (2012a), where the efficiency of the LM has been proved in order to analyze SH wave propagation in FG phononic crystals and a brief review on the methods applied to simulate wave motion in layered composites is given. The approaches have been applied to investigate time-harmonic elastic in-plane shear waves propagating in periodically laminated composites with functionally graded interlayers.

Functionally graded (FG) phononic crystals may have elastic properties varying continuously through advanced fabrication technologies such as sputtering, pressing, sintering etc. Besides, a continuous elastic property may appear because of the diffusion processes in the processing of dissimilar layers. Wu et al. (2009) studied the propagation of elastic waves in 1D phononic crystals with FGMs varying with a power-law function. They used the spectral finite element and the transfer matrix methods as main tools to analyze band-gaps and to investigate the effects of various parameters on the wave band-gaps in FG inter-layers structures. Two different power laws were used to describe the property variation of the FG interlayers within the unit-cell, and in conjunction with the transfer matrix method the wave reflection and transmission, band-gaps were investigated. More recently, Su et al. (2012) studied the influences of the material parameters, material composition, and geometrical parameters on the band-gaps of 1D FG phononic crystals by using the plane-wave expansion method. Golub et al. (2013) analyzed the wave propagation in FG treated by recursion relations and effective boundary conditions.

In our previous work (Golub et al., 2012a), SH wave propagation in layered FG elastic phononic crystals has been investigated. In this paper we extend and further develop our methods for efficient and accurate modeling of in-plane P- and SV-wave propagation in FG periodic laminates by using the explicit FG and the multilayer models. Effects of the geometrical and material parameters of the FG phonic crystals on the wave transmission and band-gaps are analyzed in details. The extension of the transfer matrix method to the considered in-plane wave propagation problem is not straightforward due to the numerically accumulating error arising during matrix multiplications. In order to solve this problem a semi-analytical representation for the transmission coefficient is derived. The analysis of the asymptotics of the semi-analytical representation shows different types of band-gaps and gives a criteria for stop-band calculations using eigenvalues of the transfer matrix for a unit-cell.

2. In-plane wave propagation in a layered periodic structure

2.1. Statement of the problem

The propagation of plane time-harmonic elastic P- and waves in a periodically layered media or phononic crystal (PnCr) composed of N identical elastic unit-cells between two identical elastic half-planes is considered. The Cartesian coordinates (x, z) are introduced in such a way that the x-axis is parallel to the interfaces of the phononic crystal and the origin of the coordinate system is on the lower boundary of the structure (Fig. 1). Each of the N unit-cells is composed of two isotropic elastic layers (A and B) and two functionally graded (FG) interlayers between them so that the elastic properties of the whole unit-cell are continuous (Fig. 2(a)). The property variation in the local Cartesian coordinate system of the kth unit-cell of thickness H parallel to the x-axis is described by the following functions as shown in Fig. 2(c)

$$P(z) = \begin{cases} P_A, & z \in [0, h_A], \\ (P_B - P_A)\left(\frac{z-h_A}{h_F}\right)^n + P_A, & z \in [h_A, h_A + h_F], \\ P_B, & z \in [h_A + h_F, h_A + h_F + h_B], \\ (P_B - P_A)\left(\frac{H-z}{h_F}\right)^n + P_A, & z \in [h_A + h_F + h_B, H]. \end{cases} \quad (1)$$

Here the function $P(z)$ denotes an appropriate material property with P_A and P_B being the boundary values corresponding to the mass density or the Lamé constants of the materials A and B (density ρ_A, ρ_B or Lamé constants λ_A, λ_B and μ_A, μ_B), h_A and h_B are the thicknesses of the homogenous layers, h_F is the thickness of the