International Journal of Solids and Structures 51 (2014) 2518–2528

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A study of different modes of collapse in metallic hemispherical shells resting on flat platen and compressed with hemispherical nosed indenter

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ARTICLE INFO

Article history:
Received 7 December 2013
Received in revised form 27 February 2014
Available online 1 April 2014

Keywords:
Hemispherical shells
Rolling plastic hinge
Modes of collapse
Quasi-static testing
Energy absorption
Aluminium

ABSTRACT

The paper presents experimental and analytical studies on axial compression of aluminium spherical shells having Radius/wall thickness (R/t) ratios between 23 and 135. Quasi-static compressive load was applied centrally and with offset through a indenter having diameter of 22 mm. Testing was carried out on an INSTRON machine having 250 T capacity. Shells having different radius and wall thickness were tested, to classify their modes of collapse and their corresponding energy absorption mechanism. In experiments shells of lower R/t values were found to collapse due to formation of an inward dimple associated with a rolling plastic hinge in central as well as in offset loading. On the other hand, shells of higher R/t values were collapsed initially with formation of an axisymmetric inward dimple, but in later stage of compression showed buckling of non-symmetric shape consisting of integral number of lobes and stationary plastic hinges. The stationary hinges were formed between consecutive lobes. Experimental observations are used to propose an analytical model for prediction of load–compression and energy–compression curves. The results obtained from analytical model compared with the experimental results and found match fairly well.

1. Introduction

Metallic shell elements such as cylindrical shells, conical shells, and domes are frequently employed as energy absorbing elements in crashworthiness applications. Furthermore these shell elements are also employed in space vehicles, submarines, buildings and storage tanks. In the last four decades many researchers (Johnson and Reid, 1978; Updike, 1972; Kitching et al., 1975; Calladine, 1986; De Oliveira and Wierzbicki, 1982; Kinkead et al., 1994; Blachut and Galletly, 1995; Gupta et al., 1998, 1999, 2008, 2001; Gupta and Gupta, 2006, 2009, 2013; Gupta, 2008; Ruan et al., 2006; Karagiozova et al., 2012; Shahin and Hayder, 2012a,b) have studies the collapse mechanics of these elements under different types of loadings including both quasi-static and dynamic loadings. The studies were carried out by conducting experimental and analytical investigations. Johnson and Reid (1978) reviewed the different modes of collapse of different thin-walled shells and also their corresponding load–compression curves.

Among the different shell elements the hemispherical shell is able to resist higher pure internal pressure loading than any other shell element having the same wall thickness and radius. The hemispherical shell is also a major component of pressure vessel construction. In practical situation, generally the pressure vessels are subjected to external loading due to hydrostatic pressure, or external impact in addition to internal pressure. Therefore, these should be designed to resist the worst combination of loading without failure. The load transmitted by a hemispherical nosed indenter applied at the apex of the hemispherical shell is considered a common external load. Therefore, study on the initial buckling and plastic buckling propagation of hemispherical shells is also important.

Different modes of collapse of metallic hemispherical shell between two flat rigid plates were investigated by a small number of researchers (Updike, 1972; Kitching et al., 1975; Calladine, 1986; De Oliveira and Wierzbicki, 1982; Kinkead et al., 1994; Blachut and Galletly, 1995; Gupta et al., 1998) by performing experiments and proposing analytical models using their experimental findings.

Calladine (1986) proposed solution of axial compression of shells in which compression was taken up to several times of the

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http://dx.doi.org/10.1016/j.ijsolstr.2014.03.023
0020-7683/

Nomenclature

R	mean radius of the spherical shell
L	span of the spherical shell
Z	depth of the spherical shell
t	average thickness of the spherical shell
r	mean radius of the parallel circle at any point of compression
r_p	radius of the rolling or travelling plastic hinge
h	total axial compression of the spherical shell at any stage of compression
dh	incremental axial compression of the spherical shell at any stage
N	number of stationary plastic hinges
l	length of the stationary plastic hinges induced during a compression of dh
P	load on the spherical shell at any stage of compression
M_p	plastic moment
dl	incremental length of spherical shell along the meridian covered in an incremental stage of compression "dh"
d_ϕ	angle subtended at centre over the incremental length "dl"
θ	offset angle
l_mt	total meridional length of assumed hemispherical shell from apex to rim or base
l_mp	total meridional length of shell from apex to point where it flattens
e	meridian strain
dv	volume of shell in an incremental stage of compression "dh"
dA	section area in an incremental stage of compression "dh"
dW_s	work absorbed by stationary plastic hinge in an incremental stage of compression "dh"
dW_r	work absorbed by rolling plastic hinge in an incremental stage of compression "dh"
dW_m	work absorbed by meridional work in an incremental stage of compression "dh"
dW	total work absorbed in an incremental stage of compression "dh"

wall thickness but much smaller than the radius. De Oliveira and Wierzbicki (1982) investigated the mode of deformation of spherical shell deformed under concentrated point load as well as between rigid plates. Their analysis was based on the formation of two rolling plastic hinge in the current deforming region. On the basis of their analysis they proposed an equation to determine collapse load for spherical shell compressed under central point load. Load-deformation variations were proposed to be independent of the radius of spherical shell. They also understood that their solution could accommodate compression quite higher than the compression for which Calladine (1986) proposed solution.

Gupta et al. (1998) performed experiments on metallic spherical shells of R/t values ranging between 15 and 240. They found that the deformation occurs in three stages namely local flattening, inward dimpling and formation of multiple lobes. They conducted analysis by considering all these three stages of deformations. The radii of rolling plastic hinges were measured from experiments and an analytical model was developed based on the energy dissipation. In some recent papers Gupta and his associates (Gupta et al., 1999, 2008, 2001; Gupta and Gupta, 2006, 2009) employed finite element code to investigate the development of axisymmetric mode of collapse of hemispherical domes (Gupta et al., 2001; Gupta and Gupta, 2006), tubes (Gupta et al., 1999, 2008) and shells of different wall thickness (Gupta and Gupta, 2009) and combined geometry (Gupta, 2008). In the mode of collapse of spherical domes, they (Gupta et al., 2001; Gupta and Gupta, 2006) considered the development of rolling plastic hinge and compared their experimental and numerically simulated FE load–compression variations and collapse mode. They results were comparable.

Axial compression of spheres and spherical arrays and snap-through behaviour of an elastic spherical shell was studied by Yu and his associates (Ruan et al., 2006; Karagiozova et al., 2012). Ruan et al. (2006) presented an experimental and analytical study on behaviour of ping pong balls subjected to axial compression by point-load, rigid plate, rigid ball, rigid cap or double rigid balls. They identified a number of bifurcation phenomena and presented their effect on the compression force. A good agreement between the analytical predictions and the experimental results was reported. Karagiozova et al. (2012) presented experimental and numerical investigation on deformation and snap-through behaviour of a thin-walled elastic spherical shell which was a table tennis ball subjected to axial compressed under quasi-static and impact loading. In their study they estimated the influence of the dynamic effects on the compression process. They concluded that impact velocity influences the mode of deformation and as a result absorbed energy. Good agreement between the numerical and experimental findings was obtained.

Recently Shahin and Hayder (2012a,b) presented analytical, numerical, and experimental results of thin hemispherical metal shells into the plastic buckling range showing the importance of geometry changes on the buckling load. In their study the hemispherical shell were rigidly supported around the base circumference against both translations and the load was applied vertically at crown with a rigid cylindrical indenter. Initial and plastic deformations were formulated on the basis of Drucker–Shield's limited interaction yield condition. The effect of the radius of the indenter on collapse load is reported. The numerical model was proposed using ABAQUS FE code. The analytical results were compared and verified with the numerical results using ABAQUS software and experimental findings. Good agreement was observed between the load–deflection curves obtained using three different approaches. They reported that the analytical solution is specifically applicable for shells having smaller values of radius/wall thickness (R/t) generally lower than 100. The reason was the change in mode of collapse from axisymmetric to nonaxisymmetric with higher R/t values. They further elaborated that the symmetry of the propagating annular zone about the vertical axis cannot be assumed throughout the load–deflection response for large R/t ratios.

In the present work axial compression of aluminium hemispherical shells under quasi-static loading is studied. Hemispherical shells resting on flat plate are compressed with axial central point load and the offset load. On the basis of the observed experimental deformed profiles it is found that the shells deform in axisymmetric and non axisymmetric modes of collapse. Load–compression curves of the deformed specimens are studies and discussed. An analytical model of the compression process is presented for the prediction of the load–compression and energy–compression curves.