International Journal of Solids and Structures 51 (2014) 13–25

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Exact electroelastic analysis of functionally graded piezoelectric shells

G.M. Kulikov *, S.V. Plotnikova

Department of Applied Mathematics and Mechanics, Tambov State Technical University, Sovetskaya Street, 106, Tambov 392000, Russia

ARTICLE INFO

Article history:
Received 8 May 2013
Received in revised form 24 August 2013
Available online 11 September 2013

Keywords:
Functionally graded piezoelectric shell
Electroelasticity
3D exact solutions
Cross-ply and angle-ply shells
Sampling surfaces method

ABSTRACT

A paper focuses on implementation of the sampling surfaces (SaS) method for the three-dimensional (3D) exact solutions for functionally graded (FG) piezoelectric laminated shells. According to this method, we introduce inside the nth layer I_n not equally spaced SaS parallel to the middle surface of the shell and choose displacements and electric potentials of these surfaces as basic shell variables. Such choice of unknowns yields, first, a very compact form of governing equations of the FG piezoelectric shell formulation and, second, allows the use of strain–displacement equations, which exactly represent rigid-body motions of the shell in any convected curvilinear coordinate system. It is worth noting that the SaS are located inside each layer at Chebyshev polynomial nodes that leads to a uniform convergence of the SaS method. As a result, the SaS method can be applied efficiently to 3D exact solutions of electroelasticity for FG piezoelectric cross-ply and angle-ply shells with a specified accuracy by using a sufficient number of SaS.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

In the last two decades, a considerable work has been carried out on the three-dimensional (3D) exact analysis of piezoelectric laminated shells. In the literature, there are at least five approaches to 3D exact solutions of electroelasticity for piezoelectric shells, namely, the Pagano approach (Vlasov, 1957; Pagano, 1969), the state space approach, the series expansion approach, the asymptotic approach and the sampling surfaces (SaS) approach. The first four approaches are discussed and critically assessed in a survey article (Wu et al., 2008). Very recently, the SaS approach has been also applied to 3D exact solutions for piezoelectric laminated plates and shells by Kulikov and Plotnikova (2013b,c).

The functionally graded (FG) piezoelectric materials are at present widely used in mechanical engineering due to their advantages compared to traditional laminated piezoelectric materials (Birman and Byrd, 2007). At the same time, the analysis of FG piezoelectric shells is not a simple task because the material properties depend on the transverse coordinate and some specific assumptions concerning their variations in the thickness direction are required (Reddy and Cheng, 2001; Zhong and Shang, 2003). This implies that first three approaches, i.e., the Pagano approach, the state space approach and the series expansion approach cannot be applied directly to 3D exact solutions for FG piezoelectric shells. However, this becomes possible if the shell is artificially divided into a large number of individual layers (Soldatos and Hadjigeorgiou, 1990) with constant material properties through the thickness (Wu and Liu, 2007; Wu and Tsai, 2012). The use of such a technique means that 3D solutions derived are approximate. On the contrary, the asymptotic approach (Wu and Syu, 2007) and the SaS approach (Kulikov and Plotnikova, 2013d) yield the exact results because governing differential equations in both approaches are obtained through *definite integration* in the thickness direction of a shell.

This paper is intended to show that the SaS method can be also applied efficiently to 3D exact solutions of electroelasticity for FG piezoelectric laminated shells. In accordance with this method, we choose inside the nth layer I_n not equally spaced SaS $\Omega^{(n)1}, \Omega^{(n)2}, \ldots, \Omega^{(n)I_n}$ parallel to the middle surface of the shell and introduce the displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2}, \ldots, \mathbf{u}^{(n)I_n}$ and the electric potentials $\varphi^{(n)1}, \varphi^{(n)2}, \ldots, \varphi^{(n)I_n}$ of these surfaces as basic shell variables, where $I_n \geqslant 3$. Such choice of unknowns with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer leads to a very compact form of governing equations of the FG piezoelectric shell formulation. Moreover, the proposed approach gives an opportunity to utilize the strain–displacement equations, which describe exactly all rigid-body shell motions in any convected curvilinear coordinate system (Kulikov and Plotnikova, 2013a). Although the SaS method has been already applied efficiently to the exact analysis of elastic and piezoelectric shells (Kulikov and Plotnikova, 2012, 2013a,c), the application of this method to FG shells cannot be found in the current literature. Note also that an idea of using the SaS can be traced back to papers (Kulikov, 2001; Kulikov and Carrera, 2008) in which three, four and five equally spaced SaS are employed. In these contributions, the Lagrange polynomials are utilized to derive *approximate* solutions of 3D shell problems. For further information concerning the approximate solution of 3D

* Corresponding author. Tel.: +7 475 271 3299.
E-mail addresses: gmkulikov@mail.ru, kulikov@apmath.tstu.ru (G.M. Kulikov).

0020-7683/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijsolstr.2013.09.004

electroelastic shell problems the reader refers to Carrera et al. (2011) where the Legendre polynomials in the thickness direction are employed.

It is necessary to mention that the proposed approach with equally spaced SaS (Kulikov and Plotnikova, 2011) does not work properly with Lagrange polynomials of high degree because the Runge's phenomenon can occur, which yields the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equally spaced nodes is increased then the oscillations become even larger. However, the use of Chebyshev polynomial nodes inside each layer can help to improve significantly the behavior of Lagrange polynomials of high degree for which the error will go to zero as $I_n \to \infty$.

The authors restrict themselves to finding five right digits in all examples presented. To achieve a better accuracy, the more SaS inside each layer should be taken.

2. Kinematic description of laminated shell

Consider a thick laminated shell of the thickness h. Let the middle surface Ω be described by orthogonal curvilinear coordinates θ_1 and θ_2, which are referred to the lines of principal curvatures of its surface. The coordinate θ_3 is oriented along the unit vector $\mathbf{e}_3$ normal to the middle surface. Introduce the following notations: $\mathbf{e}_\alpha$ are the orthonormal base vectors of the middle surface; A_α are the coefficients of the first fundamental form; k_α are the principal curvatures of the middle surface; $c_\alpha^{(n)i_n} = 1 + k_\alpha \theta_3^{(n)i_n}$ are the components of the shifter tensor at SaS; $\theta_3^{(n)i_n}$ are the transverse coordinates of SaS inside the nth layer given by

$$\theta_3^{(n)1} = \theta_3^{[n-1]}, \quad \theta_3^{(n)I_n} = \theta_3^{[n]},$$
$$\theta_3^{(n)m_n} = \frac{1}{2}\left(\theta_3^{[n-1]} + \theta_3^{[n]}\right) - \frac{1}{2} h_n \cos\left(\pi \frac{2m_n - 3}{2(I_n - 2)}\right), \tag{1}$$

where $\theta_3^{[n-1]}$ and $\theta_3^{[n]}$ are the transverse coordinates of layer interfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ depicted in Fig. 1; $h_n = \theta_3^{[n]} - \theta_3^{[n-1]}$ is the thickness of the nth layer.

Here and in the following developments, the index n identifies the belonging of any quantity to the nth layer and runs from 1 to N, where N is the number of layers; the index m_n identifies the belonging of any quantity to the inner SaS of the nth layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n describe all SaS of the nth layer and run from 1 to I_n; Greek indices α, β range from 1 to 2; Latin tensorial indices i, j, k, l range from 1 to 3.

It is seen from (1) that transverse coordinates of inner SaS coincide with coordinates of Chebyshev polynomial nodes (Burden and Faires, 2010). This fact has a great meaning for a convergence of the SaS method (Kulikov and Plotnikova, 2012, 2013a).

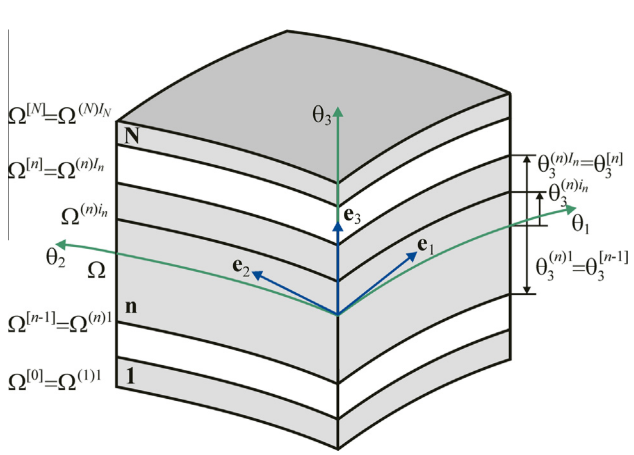

Fig. 1. Geometry of the laminated shell.

The strain tensor at SaS of the nth layer in a reference surface frame $\mathbf{e}_i$ (see, e.g. Kulikov and Plotnikova, 2013c) can be written as follows:

$$2\varepsilon_{\alpha\beta}^{(n)i_n} = \frac{1}{c_\beta^{(n)i_n}} \lambda_{\alpha\beta}^{(n)i_n} + \frac{1}{c_\alpha^{(n)i_n}} \lambda_{\beta\alpha}^{(n)i_n}, \tag{2}$$

$$2\varepsilon_{\alpha 3}^{(n)i_n} = \beta_\alpha^{(n)i_n} + \frac{1}{c_\alpha^{(n)i_n}} \lambda_{3\alpha}^{(n)i_n}, \tag{3}$$

$$\varepsilon_{33}^{(n)i_n} = \beta_3^{(n)i_n}. \tag{4}$$

Here, $\lambda_{i\alpha}^{(n)i_n}$ are the strain parameters of SaS defined as

$$\lambda_{\alpha\alpha}^{(n)i_n} = \frac{1}{A_\alpha} u_{\alpha,\alpha}^{(n)i_n} + B_\alpha u_\beta^{(n)i_n} + k_\alpha u_3^{(n)i_n} \quad \text{for} \quad \beta \neq \alpha, \tag{5}$$

$$\lambda_{\beta\alpha}^{(n)i_n} = \frac{1}{A_\alpha} u_{\beta,\alpha}^{(n)i_n} - B_\alpha u_\alpha^{(n)i_n} \quad \text{for} \quad \beta \neq \alpha, \tag{6}$$

$$\lambda_{3\alpha}^{(n)i_n} = \frac{1}{A_\alpha} u_{3,\alpha}^{(n)i_n} - k_\alpha u_\alpha^{(n)i_n}, \quad B_\alpha = \frac{1}{A_\alpha A_\beta} A_{\alpha,\beta} \quad \text{for} \quad \beta \neq \alpha, \tag{7}$$

where $u_i^{(n)i_n}(\theta_1, \theta_2)$ are the displacements of SaS; $\beta_i^{(n)i_n}(\theta_1, \theta_2)$ are the derivatives of displacements with respect to thickness coordinate at SaS given by

$$u_i^{(n)i_n} = u_i\left(\theta_3^{(n)i_n}\right), \tag{8}$$

$$\beta_i^{(n)i_n} = u_{i,3}\left(\theta_3^{(n)i_n}\right), \tag{9}$$

where u_i are the components of the 3D displacement vector in a reference surface frame $\mathbf{e}_i$, which is always measured in accordance with the total Lagrangian formulation from the initial configuration to the current configuration directly.

Now, we start with the *first assumption* of the proposed piezoelectric laminated shell formulation. Let us assume that the displacements of the nth layer $u_i^{(n)}$ are distributed through the thickness as follows:

$$u_i^{(n)} = \sum_{i_n} L^{(n)i_n} u_i^{(n)i_n}, \quad \theta_3^{[n-1]} \leqslant \theta_3 \leqslant \theta_3^{[n]}, \tag{10}$$

where $L^{(n)i_n}(\theta_3)$ are the Lagrange polynomials of degree $I_n - 1$ expressed as

$$L^{(n)i_n} = \prod_{j_n \neq i_n} \frac{\theta_3 - \theta_3^{(n)j_n}}{\theta_3^{(n)i_n} - \theta_3^{(n)j_n}}. \tag{11}$$

Using relations (9) and (10) one obtains

$$\beta_i^{(n)i_n} = \sum_{j_n} M^{(n)j_n}\left(\theta_3^{(n)i_n}\right) u_i^{(n)j_n}, \tag{12}$$

where $M^{(n)j_n} = L_{,3}^{(n)j_n}$ are the derivatives of Lagrange polynomials. The values of these derivatives at SaS are calculated as

$$M^{(n)j_n}\left(\theta_3^{(n)i_n}\right) = \frac{1}{\theta_3^{(n)j_n} - \theta_3^{(n)i_n}} \prod_{k_n \neq i_n, j_n} \frac{\theta_3^{(n)i_n} - \theta_3^{(n)k_n}}{\theta_3^{(n)j_n} - \theta_3^{(n)k_n}} \quad \text{for} \quad j_n \neq i_n,$$
$$M^{(n)i_n}\left(\theta_3^{(n)i_n}\right) = -\sum_{j_n \neq i_n} M^{(n)j_n}\left(\theta_3^{(n)i_n}\right). \tag{13}$$

It is seen that the key functions $\beta_i^{(n)i_n}$ of the laminated shell formulation are represented according to (12) as a *linear combination* of displacements of SaS of the nth layer $u_i^{(n)j_n}$.

The following step consists in a choice of the correct approximation of strains through the thickness of the nth layer. It is apparent that the strain distribution should be chosen similar to the

Download English Version:

https://daneshyari.com/en/article/277712

Download Persian Version:

https://daneshyari.com/article/277712

Daneshyari.com