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An electrically conducting interface crack with a contact zone in a piezoelectric bimaterial

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ABSTRACT

A plane problem for an electrically conducting interface crack in a piezoelectric bimaterial is studied. The bimaterial is polarized in the direction orthogonal to the crack faces and loaded by remote tension and shear forces and an electrical field parallel to the crack faces. All fields are assumed to be independent of the coordinate co-directed with the crack front. Using special presentations of electromechanical quantities via sectionally-analytic functions, a combined Dirichlet–Riemann and Hilbert boundary value problem is formulated and solved analytically. Explicit analytical expressions for the characteristic mechanical and electrical parameters are derived. Also, a contact zone solution is obtained as a particular case. For the determination of the contact zone length, a simple transcendental equation is derived. Stress and electric field intensity factors and, also, the contact zone length are found for various material combinations and different loadings. A significant influence of the electric field on the contact zone length, stress and electric field intensity factors is observed. Electrically permeable conditions in the crack region are considered as well and matching of different crack models has been performed.

1. Introduction

Active materials like piezoelectric ceramics are widely used as functional parts of many engineering systems, including sensors, transducers and actuators. However, existing micro-defects and cracks can strongly influence their behavior and reduce their strength. Very often, a crack contains air. Since the dielectric permeability of air is much less than that of piezoelectric material, the electric field inside the crack can be about 1000 times higher in magnitude than the applied remote electric field. Under such a high local electric field, an air discharge may occur inside the crack and the crack becomes a conducting one (Zhang and Gao, 2004). Also, a very soft electrode embedded in a piezoelectric matrix can often be considered as a conducting crack (Suo, 1993). Electrode stratification or electrode-matrix debonding can often lead to the appearance of conducting cracks. Therefore, studies of conducting cracks are very important for a better understanding and prediction of behaviour and failure of piezoelectric devices.

McMeeking (1987) solved the problem of an electric field around a conducting crack in dielectrics. He found that local electric field at the tip of a conducting crack is high enough. A problem of conducting crack in a homogeneous piezoelectric material was considered by, Suo (1993), Ru and Mao (1999), and Zhang and Gao (2004). For the case of electrostrictive materials, this problem was studied by Beom (1999a,b). A conducting crack between two different piezoelectric materials was considered by Beom and Atluri (2002) in the framework of the open crack model.

However, for some combinations of electromechanical loading, a crack between two different piezoelectric materials can produce essential zones of crack faces contact, which cardinally change the electromechanical fields in the whole crack region and, especially, at the corresponding crack tips. Using in such cases of the "open" crack model leads to the physically unreal overlapping of the crack faces and to the impossibility to introduce the stress intensity factors in the conventional manner. The zones of overlapping of the crack faces are usually longer than the correspondent contact zones and in some cases they can occupy more than third part of the crack length. Eliminating of these zones and determination of the real crack form and corresponding fracture mechanical parameters is the physical reason of the contact zone model consideration.

A contact zone model (Comninou, 1977; Atkinson, 1982; Dundurs and Gautesen, 1988) was developed for a crack between isotropic materials. This model was applied to interface cracks in thermopiezoelectric materials by Qin and Mai (1999) using a singular integral equation formulation and its following numerical solution. A detailed analytical investigation of an electrically permeable and electrically impermeable interface cracks with

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contact zones in a piezoelectric bimaterials has been performed by Herrmann and Loboda (2000) and by Herrmann et al. (2001), respectively. However, to our best knowledge, an electrically conducting interface crack in a piezoelectric bimaterial has not been studied yet, in spite of the possibility of appearance of large contact zones for such cracks under the action of electric field. This situation is quite different from the above mentioned cases of an electrically permeable and electrically impermeable interface crack, in which the electrical loading has only very small influence concerning the possibility of the crack faces contact.

In the present paper, we focus on a contact zone problem of electrically conducting interface crack in a piezoelectric bimaterial subjected to a tension and shear mechanical loading and an electrical field parallel to the crack faces. A significant influence of the electrical field intensity on the contact zone length and the fracture mechanical parameters is demonstrated. For the comparison the electrically permeable conditions in the crack region are considered as well.

2. General solution of the basic equation

The constitutive relations for a linear piezoelectric material in the absence of body forces and free charges can be presented in the form (Parton and Kudryavtsev, 1988)

$$\sigma_{ij} = c_{ijkl}\gamma_{kl} - e_{kij}E_k, \tag{1}$$

$$D_i = e_{ikl}\gamma_{kl} + \varepsilon_{ik}E_k, \tag{2}$$

$$\sigma_{ij,i} = 0, \quad D_{i,i} = 0, \tag{3}$$

$$\gamma_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad E_i = -\varphi_{,i}, \tag{4}$$

where u_k, φ, σ_{ij}, γ_{ij} and D_i are the elastic displacements, electric potential, stresses, strains and electric displacements, respectively; c_{ijkl}, e_{lij} and ε_{ij} are the elastic moduli, piezoelectric constants and dielectric constants, respectively. The subscripts in (1)–(4) are ranging from 1 to 3 and Einstein's summation convention is used in (1)–(3).

Substituting Eq. (4) into (1) and (2) and after that into (3), one obtains

$$(c_{ijkl}u_k + e_{lij}\varphi)_{,li} = 0, \quad (e_{ikl}u_k - \varepsilon_{il}\varphi)_{,li} = 0. \tag{5}$$

Assuming that all fields are independent on the coordinate x_2, the solution of Eq. (5), according to the method suggested by Eshelby et al. (1953), can be presented in the form (Suo et al., 1992).

$$\mathbf{V} = \mathbf{a}\, f(z), \tag{6}$$

where $z = x_1 + px_3$, $\mathbf{V} = [u_1, u_2, u_3, \varphi]^T$, $f(z)$ is an arbitrary function to be determined; $\mathbf{a} = [a_1, a_2, a_3, a_4]^T$ and p are an eigenvector and an eigenvalue, respectively, which can be obtained from the equation

$$\left[\mathbf{Q}_0 + p\left(\mathbf{R}_0 + \mathbf{R}_0^T\right) + p^2\mathbf{T}_0\right]\mathbf{a} = 0, \tag{7}$$

with 4×4 matrices $\mathbf{Q}_0$, $\mathbf{R}_0$ and $\mathbf{T}_0$ defined as

$$\mathbf{Q}_0 = \begin{bmatrix} c_{i1k1} & e_{11i} \\ e_{11i}^T & -\varepsilon_{11} \end{bmatrix}, \quad \mathbf{R}_0 = \begin{bmatrix} c_{i1k3} & e_{31i} \\ e_{13i}^T & -\varepsilon_{13} \end{bmatrix}, \quad \mathbf{T}_0 = \begin{bmatrix} c_{i3k3} & e_{33i} \\ e_{33i}^T & -\varepsilon_{33} \end{bmatrix}, \quad i,k = 1,2,3.$$

Here and afterwards, the superscript T stands for the transposed matrix.

According to Suo et al. (1992) Eq. (7) has no real eigenvalues. Therefore, we denote an eigenvalue of the relation (7) with positive imaginary parts as p_α and the associated eigenvectors of (7) as $\mathbf{a}_\alpha$ (subscript α here and afterwards takes the numerals 1–4). The most general real solution of Eq. (5) can be presented as (Suo et al., 1992)

$$\mathbf{V} = \mathbf{A}\mathbf{f}(z) + \bar{\mathbf{A}}\bar{f}(\bar{z}), \tag{8}$$

where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4]$ is a matrix composed of eigenvectors, $\mathbf{f}(z) = [f_1(z_1), f_2(z_2), \quad f_3(z_3), f_4(z_4)]^T$ is an arbitrary vector function, $z_\alpha = x_1 + p_\alpha x_3$ and the overbar stands for the complex conjugate.

Consider the vector

$$\mathbf{t} = [\sigma_{13}, \sigma_{23}, \sigma_{33}, D_3]^T. \tag{9}$$

Using Eqs. (1) and (2), this vector can be presented in the form

$$\mathbf{t} = \mathbf{B}\mathbf{f}'(z) + \bar{\mathbf{B}}\bar{\mathbf{f}}'(\bar{z}), \tag{10}$$

where the 4×4 matrix $\mathbf{B}$ is defined as

$$B_{J\alpha} = (E_{3JK1} + p_\alpha E_{3JK3})A_{K\alpha} \text{ (not summed over index } \alpha), \quad J,K = 1,2,3,4 \tag{11}$$

and $\mathbf{f}'(z) = \left[\frac{df_1(z_1)}{dz_1}, \frac{df_2(z_2)}{dz_2}, \frac{df_3(z_3)}{dz_3}, \frac{df_4(z_4)}{dz_4}\right]^T$.

For the following analysis related to the conducting crack, it is convenient to introduce the vectors

$$\mathbf{L} = [u_1', u_2', u_3', D_3]^T, \quad \mathbf{P} = [\sigma_{31}, \sigma_{32}, \sigma_{33}, E_1]^T, \tag{12}$$

where the prime means the differentiation on x_1.

Using relations (8) and (10), these vectors can be presented in the form (Loboda and Mahnken, 2011)

$$\mathbf{L} = \mathbf{M}\mathbf{f}'(z) + \bar{\mathbf{M}}\bar{f}'(\bar{z}), \tag{13}$$

$$\mathbf{P} = \mathbf{N}\mathbf{f}'(z) + \bar{\mathbf{N}}\bar{f}'(\bar{z}), \tag{14}$$

where the matrices $\mathbf{M}$ and $\mathbf{N}$ are found by means of the reconstruction of the matrices $\mathbf{A}$, $\mathbf{B}$. They have the following form

$$\mathbf{M} = \begin{bmatrix} a_{1J} \\ a_{2J} \\ a_{3J} \\ b_{4J} \end{bmatrix}_{J=1,2,3,4}, \quad \mathbf{N} = \begin{bmatrix} b_{1J} \\ b_J \\ b_{3J} \\ -a_{4J} \end{bmatrix}_{J=1,2,3,4}. \tag{15}$$

Consider now a bimaterial composed of two different piezoelectric semi-infinite spaces $x_3 > 0$ and $x_3 < 0$ having, respectively, the properties $c_{ijkl}^{(1)}$, $e_{lij}^{(1)}$, $\varepsilon_{ij}^{(1)}$ and $c_{ijkl}^{(2)}$, $e_{lij}^{(2)}$, $\varepsilon_{ij}^{(2)}$. We assume, that the vector $\mathbf{P}$ is continuous across the whole bimaterial interface. This means that the boundary conditions at the interface $x_3 = 0$ are the following

$$\mathbf{P}^{(1)}(x_1, 0) = \mathbf{P}^{(2)}(x_1, 0) \quad \text{for } x_1 \in (-\infty, \infty). \tag{16}$$

To construct the presentations, which satisfy the interface conditions (16), we use Eqs. (13) and (14) for upper and lower half-planes which can be written in the form

$$\mathbf{L}^{(m)} = \mathbf{M}^{(m)}\mathbf{f}'^{(m)}(z) + \bar{\mathbf{M}}^{(m)}\bar{\mathbf{f}}'^{(m)}(\bar{z}), \tag{17}$$

$$\mathbf{P}^{(m)} = \mathbf{N}^{(m)}\mathbf{f}'^{(m)}(z) + \bar{\mathbf{N}}^{(m)}\bar{\mathbf{f}}'^{(m)}(\bar{z}). \tag{18}$$

Satisfying the bimaterial interface condition (16), the following presentations are obtained similarly to Loboda and Mahnken (2011)

$$\langle \mathbf{L}(x_1) \rangle = \mathbf{W}^+(x_1) - \mathbf{W}^-(x_1), \tag{19}$$

$$\mathbf{P}^{(1)}(x_1, 0) = \mathbf{S}\mathbf{W}^+(x_1) - \bar{\mathbf{S}}\mathbf{W}^-(x_1), \tag{20}$$

where $\mathbf{S} = \mathbf{N}^{(1)}\mathbf{D}^{-1}$, $\mathbf{D} = \mathbf{M}^{(1)} - \bar{\mathbf{M}}^{(2)}\left(\bar{\mathbf{N}}^{(2)}\right)^{-1}\mathbf{N}^{(1)}$, $\mathbf{W}(z)$ is a vector-function which is analytic in each semi-infinite plane and $\mathbf{W}^+(x_1) = \mathbf{W}(x_1 + i\cdot0), \mathbf{W}^-(x_1) = \mathbf{W}(x_1 - i\cdot0)$. Here and afterwards,