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# A coupled elastoplastic-damage constitutive model with Lode angle dependent failure criterion

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ABSTRACT

A coupled elastoplastic-damage constitutive model with Lode angle dependent failure criterion for high strain and ballistic applications is presented. A Lode angle dependent function is added to the equivalent plastic strain to failure definition of the Johnson–Cook failure criterion. The weakening in the elastic law and in the Johnson–Cook-like constitutive relation implicitly introduces the Lode angle dependency in the elastoplastic behaviour. The material model is calibrated for precipitation hardened Inconel 718 nickel-base superalloy. The combination of a Lode angle dependent failure criterion with weakened constitutive equations is proven to predict fracture patterns of the mechanical tests performed and provide reliable results. Additionally, the mesh size dependency on the prediction of the fracture patterns was studied, showing that was crucial to predict such patterns.



## 1. Introduction

The material models that are typically used for the ductile metals may be classified according to their formulation as uncoupled and coupled. In the former the failure criterion does not affect to the constitutive relationships, whereas in the latter the accumulated damage weakens somehow the elastic moduli, the constitutive relationships or both. Several uncoupled material models with third deviatoric invariant dependent yield function and failure criterion can be found in the literature such as the published by Wilkins et al. (1980), Bai and Wierzbicki (2008) or Bai and Wierzbicki (2010). Nevertheless, such models are not the only ones that account for the third invariant dependency. Other uncoupled models with independent failure criteria also can be found. In a recent investigation, Kane et al. (2011) used a Johnson–Cook-like constitutive relationship combined with two uncoupled failure criteria: Cockcroft–Latham (Cockcroft and Latham, 1968) criterion and a Continuum Damage Mechanics (CDM) based criterion (Lemaitre, 1996). They show the effect that the Lode parameter has in the fracture strain. Conversely, the coupled models prefer to employ classical metal yield functions such and von Mises yield function and insert the third invariant in the plasticity through elastoplastic-damage coupled constitutive relationships. That is the case of the models proposed Wierzbicki and Xue (Xue, 2007; Xue and Wierzbicki, 2008) or Nahshon and Hutchinson (2008). The conclusion achieved by all the authors using all those material models is almost equivalent. The third deviatoric invariant should be introduced in the model if complex fracture patterns are to be captured.

In the recently postulated JCX model by Chocron et al. (2011), the introduction of the third deviatoric invariant in the uncoupled Johnson–Cook model was investigated. In accordance with the previously stated, one of the conclusions that such research arose was that it was not possible to obtain complex fracture patterns unless the third invariant was somehow included in the plasticity model. In the JCX material model the third invariant was added by using a Lode angle dependent function. This formulation implied a non-convex yield surface. Although it was demonstrated that it was a perfectly valid formulation, non-convex yield surfaces are unorthodox in metal plasticity. With the same background, a new formulation using a von Mises yield surface and a coupled elastoplastic-damage constitutive model is now proposed.

Since the model is intended for high strain rate phenomena and ballistic applications, the Johnson–Cook model (Johnson and Cook, 1985, 1983) is taken as a basis. The objective is to use a Lode angle dependent function only in the definition of the equivalent plastic strain to failure. The coupled elastoplastic-damage constitutive model, similar to the damage coupled Johnson–Cook model presented by Børvik et al. (2001), carries out the rest. The influence of third deviatoric invariant is then implicitly included with the help of the coupled relationship. The concept of including the third deviatoric invariant in such a way to obtain complex fracture patterns like slanted cracks, was ascertained by Xue and Wierzbicki (2009).

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An extensive experimental campaign was carried out in order to calibrate the postulated model for the precipitation hardened Inconel 718 nickel-base superalloy. The mechanical tests were performed by employing different geometries and testing techniques. Therefore, they were divided in the three following groups:

- Quasi-static tensile tests of axisymmetric smooth and notched specimens at room temperature.
- Quasi-static tensile tests of plane specimens.
- Dynamic tests of smooth axisymmetric specimens at various temperatures.

Numerical simulations of all the tests using the explicit version of LS-DYNA non-linear finite element code were carried out in order to check the validity of the model. The capability of the proposed material model in terms of reproducing fracture patterns was also studied. The effect of the mesh size on such patterns was found as crucial.

## 2. The model

### 2.1. Stress invariant representation

Any yield surface can be described by using the $\boldsymbol{\sigma}$ stress tensor invariants $I_1, I_2$ and $I_3$ (Souza et al., 2008). This representation is extremely useful, given that in most of the cases it is associated to a geometric interpretation in the principal stress space. A scalar yield function $\phi(I_1, I_2, I_3)$ represents a surface in the principal stress space $(\sigma_1, \sigma_2, \sigma_3)$. Nevertheless, a combination of another three invariants $(\sigma_H, J_2, \theta)$ is more common. Thus, the yield function can also be a function of them as:

$$\phi = \phi(I_1, I_2, I_3) = \phi(\sigma_H, J_2, \theta) \tag{1}$$

where $\sigma_H = 1/3 I_1 = 1/3\,\mathrm{tr}(\boldsymbol{\sigma})$ is the hydrostatic stress, $J_2$ is the second deviatoric stress invariant and $\theta$ is the Lode angle. The first invariant is related with the stress tensor, whereas the other two are related with the deviatoric stress tensor.

In metals, this dependency is usually neglected (Hill, 1950). Therefore, for metal plasticity the yield function can be written as $\phi(J_2, \theta)$. As an example, two of the most extensively used yield functions are depicted in Fig. 1, von Mises and Tresca yield criterion. The former is only $J_2$ dependent, whereas the latter is $J_2, \theta$ dependent.

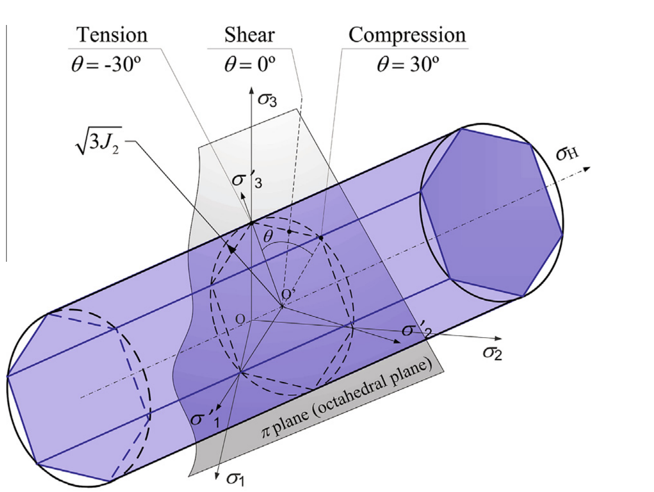


**Fig. 1.** von Mises and Tresca yield criteria. Geometric interpretation of the three invariants $(\sigma_H, J_2, \theta)$ in the principal stress space.

The second and third deviatoric stress invariants $J_2$ and $J_3$, respectively are:

$$J_2 = -I_2(\boldsymbol{\sigma}') = \frac{1}{2}\mathrm{tr}(\boldsymbol{\sigma}'^2) = \frac{1}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}' \tag{2}$$

$$J_3 = I_3(\boldsymbol{\sigma}') = \det(\boldsymbol{\sigma}') = \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma}')^3 \tag{3}$$

Typically, the third invariant is not used *per se* in the formulation of the yield function. Another invariant is used in exchange, the Lode angle. The main reason is that this last invariant has a clear geometric interpretation in the principal stress space. The Lode angle $\theta$ is:

$$\theta = -\frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}J_3}{2J_2^{3/2}}\right) = -\frac{1}{3}\sin^{-1}\left(\frac{27J_3}{2\overline{\sigma}^3}\right) \tag{4}$$

The Lode angle takes values from $-\pi/6 \leqslant \theta \leqslant \pi/6$ $(-30^\circ \leqslant \theta \leqslant 30^\circ)$, with the angle being between $\boldsymbol{\sigma}'$ and nearest pure shear line (see Fig. 1). According to the definition, $\theta = -30^\circ$ represents an axisymmetric tensile stress state, while $\theta = 30^\circ$ represents an axisymmetric compression stress state. Between them, $\theta = 0^\circ$ is for a pure shear stress state.

Since the Lode angle represents the different stress states can be also used to define the strain to failure. Most ductile fracture criteria are based on nucleation, growth and coalescence of voids. The void growth inside the material is considered to be stress triaxiality-driven. Hence, the strain to failure can also be expressed as a function of the stress triaxility $\sigma^* = \sigma_H/\overline{\sigma}$ and the Lode angle.

### 2.2. Constitutive model

The model was designated as Johnson–Cook–Xue-damage (JCXd) to distinguish it from the uncoupled version JCX postulated by Chocron et al. (2011). The details of the numerical implementation can be found in Erice (2012).

Assuming the additive decomposition of the strain tensor as:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p \tag{5}$$

the coupled elastic-damaged law reads:

$$\boldsymbol{\sigma} = w(D)\mathbf{C} : \boldsymbol{\varepsilon}^e = w(D)\mathbf{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \tag{6}$$

where $w(D)$ is the weakening function defined as:

$$w(D) = 1 - D^\beta \tag{7}$$

where $D$ is the damage parameter and $\beta$ is a material constant. $\mathbf{C}$ is the fourth-order isotropic tensor of elastic moduli given by:

$$\mathbf{C} = \left(K - \frac{2G}{3}\right)\boldsymbol{I} \otimes \boldsymbol{I} + 2G\boldsymbol{I} = \frac{\nu E}{(1+\nu)(1-2\nu)}\boldsymbol{I} \otimes \boldsymbol{I} + \frac{E}{(1+\nu)}\boldsymbol{I} \tag{8}$$

where $E$ is the elastic modulus, $\nu$ is the Poisson's ratio, $G$ is the shear modulus and $K$ is the bulk modulus.

A classical metal plasticity yield function was adopted to model the plastic flow, i.e. the von Mises yield function. For the JCXd material model the yield function is:

$$\phi(\boldsymbol{\sigma}, Y^{JCXd}) = \phi(J_2, \overline{\varepsilon}_p, \dot{\overline{\varepsilon}}_p, T, D) = \overline{\sigma}(\boldsymbol{\sigma}) - Y^{JCXd}(\overline{\varepsilon}_p, \dot{\overline{\varepsilon}}_p, T, D) \tag{9}$$

where is $\overline{\sigma} = \sqrt{3J_2}$ the equivalent stress and $Y^{JCXd}$ is the JCXd flow stress defined as:

$$Y^{JCXd}(\overline{\varepsilon}_p, \dot{\overline{\varepsilon}}_p, T, D) = w(D)Y_M(\overline{\varepsilon}_p, \dot{\overline{\varepsilon}}_p, T) \tag{10}$$