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Nonlinear viscoelastic-degradation model for polymeric based materials

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ABSTRACT

This study presents a phenomenological constitutive model for describing response of solid-like viscoelastic polymers undergoing degradation. The model is expressed in terms of recoverable and irrecoverable time-dependent parts. We use a time-integral function with a nonlinear integrand for the recoverable part and another time-integral function is used for the irrecoverable part, which is associated with the degradation evolution in the materials. Here, the degradation is attributed to the secondary and tertiary creep stages. An 'internal clock' concept in viscoelastic materials is used to incorporate the accelerated failure in the materials at high stress levels. We ignore the effect of heat generation due to the dissipation of energy and possible healing in predicting the long-term and failure response of the polymeric materials. Experimental data on polymer composites reported by Drozdov (2011) were used to characterize the material parameters and validate the constitutive model. The model is shown capable of predicting response of the polymer composites under various loading histories: creep, relaxation, ramp loading with a constant rate, and cyclic loadings. We observed that the failure time and number of cycles to failure during cyclic loadings are correlated to the duration of loading and magnitude of the prescribed mechanical loads. A scalar degradation variable is also introduced in order to determine the severity of the degradation in the materials, which is useful to predict the lifetime of the structures subject to various loading histories during the structural design stage.

1. Introduction

Polymers and polymeric composites are widely used in many engineering applications, where they are often subjected to various loading histories, i.e., constant or cyclic mechanical loads. Polymers are known as viscoelastic bodies in that they exhibit stress-relaxation (or creep deformation). The rate of stress relaxation (or the rate of creep) in a viscoelastic body is determined by the body's 'internal clock', which is associated with the movement of molecular structures of the material in response to the external stimuli (Ferry, 1961; Pipkin, 1986). This rate also changes with varying environmental conditions and mechanical loadings, as previously discussed by Schwarzl and Staverman (1952), Wineman (2002), Rajagopal and Wineman (2010) and Tscharnuter and Muliana (2013). For example, Tscharnuter and Muliana (2013) showed that increasing temperatures and strain levels in polyoxymethylene (POM) polymer accelerates the relaxation response of this polymer. Miyano et al. (2008) and Nakada and Miyano (2009) showed that the accelerated creep and cyclic responses of epoxy and various fiber reinforced polymer composites at elevated temperatures can be used to obtain long-term response of the materials. Experimental studies have also shown that polymers and polymer composites could experience creep ruptures[1] at relatively high stress levels (Raghavan and Meshii, 1997; Regrain et al., 2009; Drozdov, 2010, 2011), while at relatively low stresses, failure might not occur even after a long loading period.

There have been several experimental studies on understanding the response of viscoelastic polymers and polymeric composites under cyclic loadings that lead to fatigue failures. In many experimental studies, e.g., Sullivan (2008), Miyano et al. (2008), Cerny and Mayer (2010), Drozdov (2011) and Berer et al. (2013), etc., it was shown that during cyclic loadings strains continuously increase with time with the strain envelopes (maximum, minimum, and mean strains) follow creep-like response. Depending on the loading amplitude, frequencies, and environmental conditions, cyclic strain envelopes can experience primary, secondary and tertiary creep-like response, leading to cyclic failure. The criterion for cyclic failure is often defined in terms of number of cycles to failure, and as expected increasing loading amplitude shortens the failure time or reduces number of cycles to failures. Limited experimental studies on polymers under cyclic loading have also shown that temperature increases due to the energy dissipation

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[1] Creep response in a viscoelastic material is usually categorized as primary, secondary, and tertiary stages. Creep rupture is associated to the tertiary stage of creep, in which the rate of creep deformation increases with time and leads to failures.

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in viscoelastic materials can be pronounced even at relatively low loading amplitude (Tauchert, 1967a,b; Berer et al., 2013). This temperature generation could influence the cyclic behavior of polymers.

In order to understand the time-dependent failure in viscoelastic materials, several theoretical studies have been presented, which can be classified as: empirical model and continuum based time-dependent constitutive model that incorporates damage accumulation and microstructural characteristics. Norton power law model and its modifications are among the widely used empirical models to describe secondary and tertiary creep response of metals at elevated temperatures, in which the strain rate is defined in terms of power law function of time, $\dot{\varepsilon} = At^m$, where the parameters A and m can be adjusted to fit each creep data or these parameters A and m can be taken as functions of stress, temperature, etc. Similarly, empirical models based on power law function have been considered to determine the failure times during creep and cyclic loadings, and also number of cycles to failure in case of fatigue failure due to cyclic loading in polymers. Examples of the empirical models for creep and cyclic failures in polymer based materials can be found in Solasi et al. (2008), Miyano et al. (2008) and Guedes (2007, 2008), among others. Empirical models generally lead to rather simple mathematical expressions, which make it easier to obtain exact closed form solutions of the response of structures in the structural analysis and design processes; however they require extensive amount of testing in order to characterize the material parameters. To enhance understanding of the time-dependent failure mechanisms in viscoelastic materials, several constitutive models that incorporate damage accumulation have also been presented. Ostergren and Krempl (1979) and Cozzarelli and Bernasconi (1981) proposed a rate of damage model in order to predict failure time of materials undergoing significant creep behaviors such as metals at elevated temperatures. Similar to a yield function in plasticity, Ostergren and Krempl (1979) defined a damage function, which was called a forcing function, in order to determine the evolution of damage and they assumed that no damage progressed during unloading. They showed that rupture times and numbers of cycles to failure in the materials depend strongly on the duration of loading. Sullivan (2008) introduced a time-dependent damage function which was correlated to a reduction in load carrying capacity of the materials. Christensen (2008) incorporated a crack growth effect, which is also associated to the reduction in the strength of the materials, on determining the times to failure in the materials during creep and cyclic loadings. Drozdov (2010, 2011) formulated a viscoelastoplastic constitutive model that takes into account microstructural morphologies of the polymers to describe the creep rupture and number of cycles to failures in polymer and polymer composites. He introduced a scalar function that describes the evolution of plastic deformations in the crystalline phase of the polymers and alters the macroscopic properties of the materials. It is known that failure in the materials is accompanied by the dissipation of energy and viscoelastic materials dissipate energy. Several constitutive models have considered the effect of energy dissipation in the viscoelastic materials (Schapery and Cantey, 1966; Rajagopal and Srinivasa, 2011; Khan and Muliana, 2012).

This study introduces a time-dependent degradation phenomenological constitutive model for describing response of viscoelastic polymers undergoing various loading histories: creep, relaxation, ramp loading and cyclic loading. The model is expressed in terms of the recoverable and irrecoverable time-dependent parts. We adopt the time-integral function with a nonlinear integrand, following the nonlinear single integral and quasi-linear viscoelastic (QLV) models (Pipkin and Rogers, 1968; Fung, 1981; Muliana et al., 2013) for the recoverable part. Another time-integral function is also defined for the irrecoverable part, which is associated with the degradation formation in the materials. We use an 'internal clock' concept in viscoelastic materials in order to incorporate the accelerated failure in the materials at high stress levels. We ignore the effect of heat generation due to the dissipation of energy and possible healing in predicting the long-term and failure response of the materials. The manuscript is organized as follows. Section two discusses the constitutive model and its numerical implementation. Section three presents the material characterization and model prediction of polymer composites under a uniaxial loading. Experimental data on polymer composites reported by Drozdov (2011) were used. We also perform parametric studies on understanding the behavior of the model and present structural analyses with a primary intention to design polymer and polymer composite structural components incorporating more realistic long-term performance of the materials. Finally we dedicate section four for concluding remarks.

2. Nonlinear viscoelastic-degradation model

Response of viscoelastic polymers is manifested in simultaneous stress relaxation and creep deformation, which are attributed to the movement of macromolecular chains of the polymers. At the macroscopic level, this phenomenon is shown by a creep deformation when a constant stress is prescribed to the material; a stress relaxation when a constant strain (deformation) is maintained; a hysteretic response under cyclic loading, etc. The hysteretic behavior is attributed to the delayed response, which is also associated to the dissipation of energy, of the materials. Based on their macroscopic time-dependent response, viscoelastic materials can be classified into solid-like and fluid-like (see Wineman and Rajagopal, 2000). When subjected to external mechanical stimuli, in absence of damage/degradation, the fluid-like viscoelastic materials will continuously deform to reach a steady flow, as fluid flows, and the stress will continuously relax to zero. For the solid-like viscoelastic material, in absence of degradation, both creep deformation and stress relaxation will reach to asymptotic nonzero values. Unlike fluid-like viscoelastic materials, the solid-like viscoelastic materials will undergo complete recovery upon unloading if sufficient time is given and no damage/degradation has occurred in the materials during loadings. Macroscopic creep response in a viscoelastic material is further categorized into primary, secondary, and tertiary stages, which are based on the rates of creep deformations, as illustrated in Fig. 1a. The rate of creep deformation in the primary stage decreases with time, followed by a constant rate in the secondary stage, and the creep rate increases with time in the tertiary stage until failures. Furthermore, macroscopic response of materials depends strongly upon their microstructural characteristics and changes in their microstructural morphologies during loadings. For examples, response of polymers depends on their macromolecular structures (amorphous, crystallines, or semicrystalline). When subjected to mechanical loadings, the polymers undergo microstructural changes, i.e., polymer network rearrangement, entanglement, scission, crazing, etc.

Before developing constitutive models that describe time-dependent behaviors of viscoelastic materials, it is necessary to understand a complete behavior of these materials under various loading conditions and place them into certain classifications. The proper models, with appropriate material parameters, should be able to reasonably predict the response of materials under various loading histories. In this study, we rely on the macroscopic response of viscoelastic materials under various mechanical loading histories without any detailed information about the microstructural changes during loading. We formulate a nonlinear phenomenological constitutive model for *solid-like viscoelastic materials* incorporating three stages of creep behaviors. The solid-like