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Investigations on vibration characteristics of two-layered piezoceramic disks

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ABSTRACT

This paper investigates the transverse and planar vibration characteristics of two-layered piezoceramic disks for traction-free boundary conditions by theoretical analysis, finite element numerical calculation, and experimental measurements. Amplitude-fluctuation electronic speckle pattern interferometry (AF-ESPI), laser Doppler vibrometer (LDV), and impedance analysis were used to perform measurements and verify the theoretical solutions for extensional, tangential, and transverse vibrations. The poling direction of piezoelectric elements determines whether they are denoted as either of series- or parallel-type. This study observed that the resonant frequencies and mode shapes of the series- and parallel-type piezoceramic disks present different dynamic characteristics in resonance. Planar and transverse vibrations are coupled in series-type piezoceramic disks and uncoupled in those of parallel-type. Good agreements of dynamic characteristics determined by theoretical analysis, experimental measurements, and numerical calculation are presented for series- and parallel-type piezoceramic disks.

1. Introduction

Miniaturized piezoelectric elements are widely used as sensors and actuators in nondestructive testing devices, electro-optic modulators, and electro-acoustical equipment. Recently, high efficiency piezoelectric energy harvesters have been applied to low power electronic devices (Rödig and Schönecker, 2010). In the design of such transducers, ensuring good electromechanical transfer ability requires detailed specifications regarding vibration characteristics, and piezoceramic disks have proven valuable in this regard. The radial, tangential, and transverse vibrations of single-layer piezoceramic circular plates were analyzed using the separation of variables method and the finite element method (FEM), and the corresponding resonant frequencies were in good agreement with theoretical predictions (Huang et al., 2004). Many applications use piezoelectric disks or rings mounted on metal or fiber reinforced composite as a unimorph piezoceramic structure. A piezoelectric disk combination with a glass mirror was designed to produce different vibration modes on a flexible mirror using segmented electrodes. The vibration contours obtained by double-exposure holographic interferometry were used to determine the relationship between excited voltage and vibration displacement (Steinhaus and Lipson, 1979). Unimorphs (a combination of piezoelectric polymer and Plexiglas) have been used to simulate the full-field voltage and displacement contours of vibration modes using finite element analysis (Tseng and Liou, 1996). The static and dynamic electro-elastic modeling of annular piezoceramic disk transducers were analyzed to calculate the resonant frequencies, anti-resonant frequencies, and electromechanical coupling coefficient in extension and flexure couple vibrations. The results of theoretical analysis were compared with the experimental measurements obtained by impedance analysis (Chang et al., 1999). Noncontact ultrasonic transportation was compounded by a single-layered piezoelectric disk with segmented electrodes and a circular plate, which formed a unimorph structure to operate in flexural vibrations. Its vibration displacement and sound pressure were measured on the radial and thickness directions using a scanning laser Doppler vibrometer (LDV) to validate FEM predictions (Koyama and Nakamura, 2010). The FEM model of the flexural vibration mode used to analyze the piezoelectric unimorph had a barrel structure. The modeling design for the size in FEM results was compared with the experimental measurements of the resonant frequencies and flexural vibration modes obtained by scanning LDV (Wang et al., 2009). Although unimorphs have proven useful in the analysis and design of devices, piezoelectric annular transducers based on tri-layer piezoelectric bimorphs or combinations of dual-layer piezoceramic disks are more often used to achieve relatively large mechanically vibrating deformations. This paper studies thoroughly the vibration characteristics of two-layered piezoceramic disks for completely free boundary condition using theoretical analysis, finite element numerical calculation, and experimental measurements.

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Multilayer piezoelectric devices are often used in precision actuators and positioned control. Two or three-layered piezoelectric ceramics (often called bimorphs) provide a high stiffness and relatively large mechanical deflection. Huang and Huang (2010) derived the equivalent circuit related to the flexural vibration of five-layer piezoelectric disks according to Hamilton's principle, and discussed the voltage gain and efficiency of piezoelectric transformers operating at low frequencies. Usually, piezoelectric ceramics can be divided into two categories: series and parallel. In series-type (symmetric) devices, the upper and lower layers of piezoceramics have opposite poling direction; in parallel type (asymmetric) devices, the poling directions are the same. Lee et al. (2000) analyzed the extensional and flexural vibrations of asymmetric piezoelectric two-layered bimorph disks of unequal thickness and poling direction, and calculated the resonant frequencies, distribution of displacement, surface charge, and static response. The relationship between maximum displacement and frequency bandwidth of the clamped piezoelectric disks for unimorphs and bimorphs was investigated analytically and using FEM in order to understand the effect of size and the segmented parts of the different poling piezoceramics (Papila et al., 2008). A theoretical model was proposed for the analysis of series-and parallel-type piezoelectric annular bimorphs under free-free and free-clamped boundary conditions using admittance and impedance matrices (Ha and Kim, 2001). However, there are few results available in the literature for the full-field measurement of vibration modes for multilayered piezoceramics.

The high electromechanical transfer efficiency of parallel-type piezoelectric bimorphs has led to their wide application, and the analysis of dynamic characteristics has mostly been limited to these parallel devices. Theoretical analysis and FEM were performed for a parallel-type piezoelectric circular bimorph, and impedance analysis was used to experimentally verify the results for clamped outer rim boundary conditions (Dobrucki and Prichnicki, 1997). The analysis and optimization of the vibration behavior of the parallel-type piezoceramic rings were studied using Kirchhoff–Love plate theory and electroelastic theory (Chang and Lin, 2003). Using impedance matrices, Kim and Ha (2003) designed an ultrasonic motor operating in circumferential vibration mode for the parallel-type piezoelectric disks with segmented electrodes. The series-type piezoelectric bimorph was applied through an electrical connection and the advantages have been addressed in the literature. When operated in a coercive electric field, series-type piezoelectric bimorphs usually display larger displacement and higher reliability than parallel type (Hayashi et al., 1991). In our previous study, we investigated the three-dimensional coupled vibration characteristics of parallel- and series-type piezoelectric rectangular bimorphs using parallel and series electrical connections by experimental measurements and FEM (Huang and Ma, 2009), respectively. In this study, the theoretical analyses provide the good agreements with the experimental and numerical results of resonant frequencies, mode shapes, and electromechanical coupling coefficients for two-layered piezoceramic disks.

In the literatures, the vibration characteristics of piezoelectric ceramics have mostly been investigated through theoretical analysis and FEM. Experimental results are provided by electrical impedance analysis and point-wise vibrating measurement. To obtain a more comprehensive understanding of the vibration characteristics of two-layered piezoceramic disks, this paper compares the theoretical solutions and FEM results with experimental measurements of vibration characteristics using three techniques: (1) Amplitude-fluctuation electronic speckle pattern interferometry (AF-ESPI); (2) Laser Doppler vibrometer (LDV); and (3) Impedance analysis. AF-ESPI was used to determine the resonant frequencies and corresponding mode shapes, and present the interferoimages as fringe patterns. LDV was used to determine the resonant frequency of transverse vibration for two-layered series- and parallel-type piezoceramic disks; however, the extensional vibration characteristics, including resonant frequency and mode shape were determined using AF-ESPI. The resonant frequencies of extensional vibration modes were also verified using impedance analysis. Finite element analysis was performed using the commercial software package, ABAQUS. We obtained good agreements between experimental measurements and numerical results related to mode shape and resonant frequency. Theoretical solutions were also analyzed for extensional, tangential, and transverse vibrations and compared with the experimental and numerical results for the series- and parallel-type piezoceramic disks.

2. Theoretical analysis for two-layered series- and parallel-type piezoceramic disks

The geometrical configuration of two-layered series-type and parallel-type piezoceramic disks in series electrical connections are shown in Fig. 1. The origin of the cylindrical coordinates (r, θ, z) is located in the center of the disks. In series-type piezoceramic disks, the upper- and lower-layer piezoceramics have opposite poling directions; in parallel-type disks, the poling directions are the same. The radius and the thickness of each piezoceramic layer are denoted R and h, respectively, and the piezoceramic disks are assumed to be thin (i.e. $h \ll R$). In the theoretical analysis, we adopted the system of governing equations and hypotheses to determine the vibration characteristics of the two layered piezoceramic disk presented by Rogacheva (see Rogacheva, 1994). To derive the vibration characteristics for one piezoceramic disk with crystal symmetry class C_{6mm}, the piezoelectric d-form linear constituent equations are:

$$\begin{bmatrix} e_{rr} \\ e_{\theta\theta} \\ e_{zz} \\ e_{\theta z} \\ e_{zr} \\ e_{r\theta} \\ D_r \\ D_\theta \\ D_z \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & d_{32} \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 & 0 & 0 & d_{33} \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & 0 & s_{44}^E & 0 & d_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{15} & 0 & \varepsilon_{11}^\sigma & 0 & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 & 0 & \varepsilon_{11}^\sigma & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^\sigma \end{bmatrix} \begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{\theta z} \\ \sigma_{zr} \\ \sigma_{r\theta} \\ E_r \\ E_\theta \\ E_z \end{bmatrix} \quad (1)$$

where e_{ij}, σ_{ij}, E_i, and D_i $(i, j = r, \theta,$ and $z)$ are the components of strain, stress, electric field, and electric displacement, respectively. The s_{pq}^E $(p, q = 1 - 3)$ are the compliance constants under a constant electric field, d_{pq} are the piezoelectric constants, and ε_{pq}^σ are the

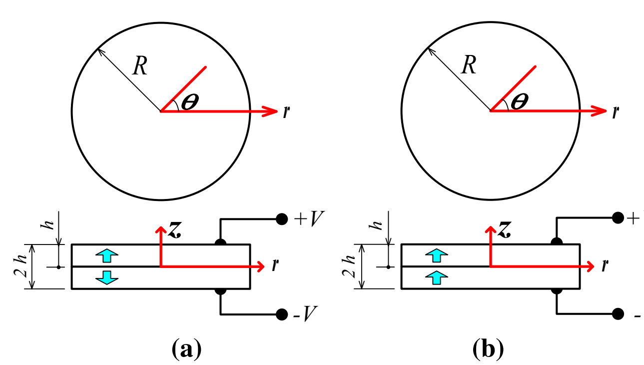

Fig. 1. Geometry and coordinate system of the two-layered piezoceramic disks: (a) series type in series electrical connection and (b) parallel type in series electrical connection.