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Strength homogenization of matrix-inclusion composites using the linear comparison composite approach

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ABSTRACT

A homogenization procedure to estimate the macroscopic strength of nonlinear matrix-inclusion composites with different strength characteristics of the matrix and inclusions, respectively, is presented in this paper. The strength up-scaling is formulated within the framework of the yield design theory and the linear comparison composite (LCC) approach, introduced by Ponte Castaneda (2002) and extended to frictional models by Ortega et al. (2011), which estimates the macroscopic strength of composite materials in terms of an optimally chosen linear thermo–elastic comparison composite with a similar underlying microstructure. In the paper various combinations for the underlying material behavior for the individual phases of the composite are considered: The matrix phase can be a quasi frictional material characterized either by a Drucker–Prager-type (hyperbolic) or an elliptical strength criterion, which predicts a strength limit also in hydrostatic compression, while the inclusion phase either may represent empty pores, pore voids filled with a pore fluid, rigid inclusions, or solid inclusions, whose strength characteristics also maybe described by a Drucker–Prager-type or an elliptical strength criterion. For generating the homogenized strength criterion efficiently in such general cases of matrix-inclusion composites, a novel algorithm is proposed in the paper. The validation of the proposed strength homogenization procedure for selected combinations of strength characteristics of the matrix material and the inclusions is conducted by comparisons with experimental results and alternative existing strength homogenization models.

1. Introduction

The macroscopic properties of materials characterized by a heterogeneous microstructure, such as natural or artificial composite materials (concrete, geological materials, fibre reinforced composites) is governed by the properties, the shape and topology of the individual components (generally denoted as material phases) related often to a large range of spatial scales. For the determination of macroscopic properties of heterogeneous materials on the basis of the knowledge of their microstructure, appropriate multiscale methods are required. Such methods may be based upon computational multiscale methods or on analytical methods such as continuum micromechanics. Computational multiscale methods are attempting to directly numerically resolve the meso- or microstructure of heterogeneous materials by means of numerical discretization methods such as the finite element method and generate macroscopic quantities from homogenization over the subscale model (the representative elementary volume). For a survey of this class of multiscale methods we refer to Hain and Wriggers (2006), Sun et al. (2011) and Fish and Wagiman (1993). While this class of methods evidently allows a detailed analysis of the interactions between phases at lower scales, its computational effort is enormous. In cases, when homogenized properties, such as macroscopic elastic stiffnesses, viscosities, permeabilities or material strength are required based upon local information from the different phases (this task will be denoted in the following as "upscaling"), analytical methods may serve as a powerful conceptual basis. As far as the upscaling of linear properties is concerned, continuum micromechanics provides a well established framework. By now classical homogenization models are available for the homogenization of elastic properties (e.g. Zaoui, 2002; Dormieux et al., 2006), electrical conductivity (e.g. Hermance, 1979; Torquato, 1985), and, more recently, for diffusion properties (Dormieux and Lemarchand, 2001, Lemarchand et al., 2003, Pivonka et al., 2004, Scheiner et al., 2008) and elastic viscosities (e.g. Friebel et al., 2006; Sanahuja, 2013).

In contrast, the determination of strength properties of heterogeneous materials, due to the nonlinear nature of the mechanical principles that underly strength properties, still remains a challenge. Among the rare contributions, earlier methods for strength

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homogenization were mainly based on limit analysis that provides estimates for the dissipation at plastic collapse by employing the lower and upper bound theorems of yield design (e.g. Melan, 1936; Salencon, 1990). By solving a yield design boundary value problem, the strength capacity of various highly idealized composite materials, such as fiber reinforced composites (de Buhan and Taliercio, 1991), and fluid-saturated porous materials (de Buhan and Dormieux, 1999) can be determined. An upscaling scheme based on numerical limit analysis was presented in Fuessl et al. (2008) for the determination of strength envelopes of porous materials, taking localized material failure into account. An alternative approach was proposed and improved by Ponte Castaneda (1991, 1996, 2002), which is characterized by the use of optimally chosen, so-called "linear comparison composites" (LCC) to deliver estimates for the effective mechanical properties of porous and rigidly reinforced composites, that are exact to second-order in the heterogeneity contrast.

Barthelemy and Dormieux (2003, 2004) and Maghous et al. (2009)) have proposed an analytical approach for the strength homogenization of cohesive–frictional matrix materials with pores or rigid inclusions. The main underlying idea of this approach is to replace the corresponding limit analysis by a sequence of visco-plastic problems. For the resulting homogenized properties at the limit stress or strain state the modified secant method is used. The model has been applied for the prediction of the macroscopic strength of highly filled composite materials, such as cement-based mortars, for which the friction coefficient of the composite is higher than that of the matrix (Lemarchand et al., 2002; Heukamp, 2005). Alternatively, Pichler and Hellmich estimate the stiffness and strength of cement paste through an elastic limit analysis, since in particular for the cement paste, the elastic limit of hydrate govern the overall elastic limits (Pichler et al., 2009; Pichler and Hellmich, 2011).

More recently, Ortega et al. (2011) have developed a strength homogenization method for cohesive–frictional materials affected by the presence of porosity and rigid-like inclusions. Within the framework of the yield design theory (Salencon, 1990) the linear comparison composite approach (Ponte Castaneda, 2002; Lopez-Pamies and Ponte Castaneda, 2004) has been extended from the application of nonlinear hyper-elastic composites to elasto–plastic matrix-inclusion composites, allowing consideration of the frictional behavior of the matrix material in case that it may be represented by means of a Drucker–Prager-type strength criterion.

In this paper, the LCC method is adopted to investigate the applicability of this approach for more general classes of heterogeneous materials such as cementitious or geological materials consisting of different material phases, such as aggregates or pores. In addition to an idealization as two-phase porous materials, characterized by a solid matrix and pores either filled by air or by water, also three phase composites, in which additional solid inclusions are embedded within the solid matrix, are considered in the homogenization approach. More specifically, the matrix phase is considered as a cohesive–frictional material represented either by a Drucker–Prager-type (hyperbolic) strength criterion or an elliptical strength criterion, which predicts a strength limit also in hydrostatic compression. In the case of solid (deformable) inclusions, their strength characteristics are also assumed to be described either by a Drucker–Prager-type or an elliptical strength criterion, however, with different strength properties as compared to the matrix.

The remainder of the paper is structured as follows: Section 2 recalls the theoretical background of the adopted LCC method. In Section 3 a detailed description of the implementation of the LCC methodology for matrix-inclusion composites is presented, followed by the application to the above-mentioned combinations of matrix-inclusion morphologies in Section 4. To this end, a novel efficient algorithm is proposed to generate the homogenized strength criterion in Section 4. The resulting macroscopic strength envelopes obtained for selected scenarios for nonlinear composites are validated in Section 5 by means of comparisons with experimental results and with analytical estimations obtained from other strength homogenization models.

2. Theoretical background

Within the framework of yield design theory (Salencon, 1990), we adopt the strength homogenization method proposed by Ortega et al. (2011) based on the application of the LCC theory (Ponte Castaneda, 2002). In order to motivate the forthcoming developments, we recall briefly the elementary concepts of the yield design theory and the LCC approach.

2.1. Upper bound theorem and yield design

The problem of strength homogenization of a composite material composed of different material phases is framed within the yield design theory, with the focus of determining the macroscopic dissipation capacity through limit analysis. The lower bound theorem based on statically and plastically compatible stress states underestimates the actual dissipation capacity, whereas the upper bound theorem associated with a kinematically compatible velocity field satisfying the normality rule of plastic flow overestimates it. The upper bound theorem is generally preferred against to the lower bound theorem, because the kinematically compatible velocity field is easier to find than the statically admissible stress field (Ulm and Coussy, 2003, chap. 9).

We consider a composite material composed of different material phases characterized by a smaller length scale as compared to the scale of a representative elementary volume (REV) of the composite. Considering the properties of the individual phases on the grain size level (i.e. the scale of the individual inclusions, denoted in the sequel as "micro-scale"), the strength characteristics of a material phase i within the composite is assumed to be characterized by an individual convex failure criterion expressed in terms of the CAUCHY stress tensor $\boldsymbol{\sigma}$ at the micro-scale:

$$\mathcal{F}_i[\boldsymbol{\sigma}] \leqslant 0 \quad \iff \quad \boldsymbol{\sigma} \in G_i. \tag{1}$$

G_i denotes the convex domain of admissible microscopic stress states. Accordingly, at plastic collapse the maximum dissipation capacity of the material phase is defined by the support function π_i of G_i

$$\pi_i[\boldsymbol{d}] = \sup_{\boldsymbol{\sigma} \in G_i}\{\boldsymbol{\sigma} : \boldsymbol{d}\}, \tag{2}$$

where $\boldsymbol{d}[\boldsymbol{v}]$ is the strain rate corresponding to the velocity field $\boldsymbol{v}$, and 'sup' denotes the supremum, or least-upper bound, of the set G_i. For a given value of $\boldsymbol{d}$, the condition $\boldsymbol{\sigma} : \boldsymbol{d} = \pi_i[\boldsymbol{d}]$ defines a hyperplane $\mathcal{H}[\boldsymbol{d}]$ in the stress space, which is tangent to the boundary ∂G_i of the admissible stress domain G_i at the stress point $\boldsymbol{\sigma}$, where $\boldsymbol{d}$ is normal to ∂G_i (see Fig. 1). This is the so-called dual definition of the strength domain G_i under the condition of associated plasticity (Ulm and Coussy, 2003), i.e. G_i can be defined either through the failure criterion $\mathcal{F}_i$ or the support function π_i.

The main purpose of the yield design approach is the evaluation of the macroscopic support function Π_{hom}, and the determination of the macroscopic stress $\boldsymbol{\Sigma}$ at the boundary of the macroscopic strength domain ∂G_{hom}. For a given macroscopic strain rate $\boldsymbol{D}$, which is the average of the microscopic strain rate $\boldsymbol{d}$ over the domain Ω occupied by the composite,

$$\boldsymbol{D} = \overline{\boldsymbol{d}} = \frac{1}{|\Omega|}\int_{\Omega} \boldsymbol{d}\,\mathrm{d}\Omega, \tag{3}$$