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# Impact of contact stiffness heterogeneities on friction-induced vibration

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ABSTRACT

It is well-known that the occurrence of squeal depends on numerous phenomena still relatively unknown. Furthermore, squeal is affected by several factors on both the micro and macro scales. The present paper proposes some potential interactions between stiffness heterogeneities of the contact surface and the potential for triggering the squeal.

Consequently, an analytical model has also been developed to highlight the impact of certain parameters on the squeal phenomenon. The contact surface has been modeled by connecting the disc with the pad via distributed springs (contact stiffness). From this model, a static equilibrium and a complex modal analysis were performed to determine respectively the pressure distribution and the natural frequencies of the system.

In this configuration, the results demonstrate that the introduction of heterogeneities could change the dynamic behavior of the system. Moreover, the influence of the size of the heterogeneities was studied and results show that this parameter had an influence on the likelihood of squeal occurrence. The paper also points out a heterogeneities correlation length that is reasonable to consider for this special configuration. Such information is relevant when interpreting the occurrence of noise as a function to the sizes of the components of the brake pad and the morphology of the surfaces in contact.



## 1. Introduction

In the transport industry, there sometimes occurs a loud noise or high-pitched squeal as the brakes are applied. The latter can be of great discomfort to the human ear because of its high frequency of greater than 1 kHz (Akay, 2002). The origin of this unwanted noise is issued from unstable dynamic behavior of the brake system due to frictional forces.

During the past few decades, a significant number of theories have been developed regarding this instability phenomenon involving friction-induced vibration: stick-slip (Oden and Martins, 1985), negative friction coefficient sliding velocity slope (Mills, 1938), sprag-slip (Spurr, 1961) and mode lock-in (North, 1976). Stick-slip comes about when there are friction issues, sprag-slip is related to geometric effects from coupling between systems with different degrees of freedom, and mode lock-in is caused by self-excited vibrations. In numerous cases of squeal in braking systems, the instability is linked to the mode lock-in mechanism (Jarvis and Mills, 1963). These different theories are associated to several numerical models which can enable to predict the squeal phenomenon. On the one hand, a model based on the real geometry using a finite element method has been developed to investigate the effects of system parameters, such as the coefficient of friction between the pad and the disc (Trichês et al., 2008), or the stiffness of the disc (Liu et al., 2007) etc. On the other hand, minimal models or low-order finite element models have been used to gain understanding of the role of each parameter, essentially with an analytical approach (Hoffmann et al., 2002), or for control (Massi et al., 2006). Even though these models are relatively well understood at the system scale, all the studies cited above have been carried out with homogeneous and mostly isotropic materials. However, it is well-know that braking depends on multi-scale and multi-physics phenomena which are still unknown to a large extent. Indeed, squeal is an interdisciplinary issue involving dynamics, tribology, acoustics, etc. largely conducted separately. Furthermore, squeal is affected by many different factors on both the micro and macro scales. Phenomena at small scales when it comes to both length (microscopic contact effects) and time (high-frequency vibrations), affect and are affected by phenomena at large scales (wear, behavior of the tribological triplet and dynamics of the entire brake system). Nevertheless, studies do not necessarily demonstrate the impact of the various scales (third body, bodies

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in contact, components, heterogeneities etc.) on the likelihood of squeal occurrence.

Nevertheless, various brake studies take into account heterogeneities using mainly single-scale approaches to describe wear, damage etc. For example, at the third body scale, various methods enable to describe the surface dynamics during friction and wear and can show the formation of self-affine surfaces as found in modeling of real surfaces such as with the discrete element method (Nguyen et al., 2009; Renouf et al., 2011; Richard et al., 2008) or cellular automata (Popov and Psakhie, 2007; Dimitriev et al., 2008; Wahlstrom et al., 2011). An other contact scale which can find concerns heterogeneities introduced by machining defaults, mounting defects, etc. To model this scale, one of the first models was that proposed by Greenwood and Williamson (1966) in which the elastic contact between a rigid plane and spherical asperities was considered by modifying the contact stiffness. These statistical models have been enriched by many authors, e.g., (Bush et al., 1992; Whitehouse and Archard, 1970) and provide fast and accurately responses.

The deterministic approaches have been developed to introduce a better geometrical description using mathematical functions to represent the asperities. Elastic, perfectly plastic (Chang et al., 1987) or elastoplastic behaviors (Zhao et al., 2000) as well as interactions between asperities (Zhao and Chang, 2001) have been integrated in models. Also from a geometrical viewpoint, but at a larger scale, Bonnay et al. (2011) took into account a thickness variation of the disc and showed the influence of this perturbation on the mode lock-in. Another scale of heterogeneities concerns the components presented in the friction pad. Indeed, modern brake pads are fabricated from composites comprising many different components (structural materials, matrix, filler and frictional additives).

These heterogeneities play a key role for the local behavior through the local properties of each component (stiffness, bulk modulus etc.). Mbodj et al. (2010) and Peillex et al. (2008) studied the mechanical properties of the heterogeneities and considered the pad material using a homogenization technique while (Alart and Lebon, 1998) mixed static frictional contact and heterogeneous materials using an asymptotic homogenization technique. Recently, Magnier et al. (2011) proposed an analytical model with a contact interface including stiffness heterogeneities between pad and disc and demonstrated the influence of these kinds of heterogeneities on mode lock-in.

Consequently, the objective of this paper has been to present an analytical model including stiffness heterogeneity in the contact. This numerical model was a simplified pin-on-disc setup inspired by a experimental arrangement developed at the Laboratoire de Mécanique de Lille (France). Focus was put on the influence of the heterogeneities and their sizes on the likelihood of squeal occurrence via mode lock-in.

## 2. Semi-analytical model

### 2.1. Description

The semi-analytical model was inspired by an experimental setup developed in our lab. This specific experimental arrangement was simplified with a reduced number of parts to control the dynamic behavior, and the model comprised a thin plate, a pad housing, a friction pad and a disc as shown in Fig. 1(a). In the model, the disc was expressed by a vibration system with one degree of freedom (translational direction, namely $y_d$) and the pad was expressed by a vibration system with two degrees of freedom (translational and rotational directions, namely respectively $y_p$ and $\phi_p$) as shown in 1(b). This analytical model corresponds to that developed by Oura et al. (2009) with the exception that the rotation was not at the center of the pad.

In the model, $K_1$ and $K_2$ represent the $Y$-direction stiffness on each side of the thin plate. The stiffness of the friction pad was modeled by a parallel distribution of springs, where the stiffness of each spring $k_i$ was directly dependent on the material's mechanical properties, i.e., $k_i = \frac{E.S}{h_i}$ with $h_i$ is the height of each spring and S is equal to the discretization-dependent area of each spring. So, this parameter physically represent the stiffness of the bulk. Initially, the pad material was considered to be homogeneous with a Young modulus $E$ fixed to 3000 MPa. The pad was assigned a height ($h_i$) of 10 mm before deformation. Table 1 presents the parameter values that were used in the computation.

This analytical model was first computed in static equilibrium under conditions of pressure and sliding. This step rendered it possible to obtain the contact pressure distribution that was injected. This was done in the second step of the computation, in a complex eigenvalues analysis to determine the natural frequencies of the system. The dynamic equations were as follows:

$$\begin{cases} M_d \ddot{y}_d = -K_d y_d - \sum_{i=1}^{n} k_i y_i \\ M_p \frac{\ddot{y_2} - \ddot{y_1}}{2} = -K_1 \left(y_1 + \frac{y_2 - y_1}{d} x_1\right) - K_2 \left(y_2 + \frac{y_2 - y_1}{d} x_2\right) - \sum_{i=1}^{n} k_i y_i \\ J \frac{\ddot{y_2} - \ddot{y_1}}{d} = -K_1 \frac{y_2 - y_1}{d} x_1^2 - K_2 \frac{y_2 - y_1}{d} x_2^2 + \sum_{i=1}^{n} k_i y_i \mu h + \sum_{i=1}^{n} k_i y_i l \end{cases}$$

In this study, only the out-of-plane bending mode of the disc was considered. The assumed disc vibration was represented by a model with a single degree of freedom. The equivalent mass $M_d$ was set to be equal to the mass of the disc. The disc stiffness $K_d$ was calculated using the eigenfrequency determined with the finite element method and $M_d$. $M_p$ represents the total mass, including the pad and the pad-housing, and $J$ is the moment of inertia calculated at the center of mass of the plate spring. The spring coefficients, $K_1$ and $K_2$ were determined from $M_p$,$J$ and the numerical pad frequencies. These two springs enabled to model the thin plate and achieve the two pad modes. $y_1$ and $y_2$ represent the elongation of the $K_1$ and $K_2$ springs during translation of the pad, and $d$ is the distance between these two springs. $y_i$ represent the elongation of each spring $i$. All these parameters are presented in Table 2 and were inspired by the experimental set-up.

Finally, $x_1$ (resp. $x_2$) corresponds to the distance between the center of rotation of the pad and the spring K1 $K_1$ (resp. $K_2$).

After a quasi-static solution under sliding conditions, a complex eigenvalues analysis was performed and the results were written as $\lambda = \alpha + j\omega$. Here, $\lambda$ represents the eigenfrequency and $\omega$ the real part. If the real part is zero, the mode is considered as stable. Conversely, a non-zero real part indicates an unstable mode.

### 2.2. Illustration of a semi-analytical model considering a homogeneous stiffness distribution

This section illustrates how to determine the evolution of the system's eigenfrequencies in relation to the coefficient of friction and to identify the unstable configurations. Indeed, it has been well established that the coefficient of friction is a relevant parameter in instability (Massi et al., 2006). The pad-housing and the thin plate were considered to be of steel with mechanical properties set to 210000 MPa for the Young modulus and 0.3 for the Poisson coefficient. For the pad, a value of 3000 MPa was taken for the isotropic Young modulus. For the semi-analytical model, the pad contact area was set to $30 \times 20\ \text{mm}^2$ with a discretization of $300 \times 200$ elements. The extremities of the springs $K_1$ and $K_2$ were submitted to a vertical displacement of $-0.5$ mm to obtain an equivalent normal force of 300 N. In the $x$-direction, a translation of the disc was applied to simulate the slip condition.