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Numerical study of shear band instability and effect of cavitation on the response of a specimen under undrained biaxial loading

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ABSTRACT

This paper presents an extension of the local second gradient model to multiphasic materials (solids particles, air, water) and including the cavitation phenomenon. This new development was made in order to model the response of saturated dilatant materials under deviatoric stress and undrained conditions and possibly, in future, the behavior of unsaturated soils. Some experiments have showed the significance of cavitation for the hydromechanical response of materials. However, to date and as far as we are aware, no attempt was made to implement the cavitation as a phase change mechanism with a control of pore pressure. The first part of the results section explores the effects of permeability, dilation angle and loading rate on the stability of shear bands during a localization event. The reasons underlying the band instability are discussed in detail, which helps defining the conditions required to maintain stability and investigating the effects of cavitation without parasite effect of materials parameters or loading rate. The model showed that, if a uniform response is obtained, cavitation triggers localization. However, in case of a localized solution, cavitation follows the formation of the shear band, with the two events being quite distinct.

1. Introduction

Like many engineering materials, geomaterials can exhibit elastic and/or inelastic deformations depending on the stress state they are subjected to. While isotropic compression leads to contraction, a deviatoric stress can result in contraction or dilation, depending on the initial density of the soil. Experimental research has shown that the volumetric deformations in specimens subjected to deviatoric stress (typically by triaxial or biaxial loading) are not necessarily uniform: shear bands can develop and, in them, takes place most of the inelastic volume changes. The formation of shear bands is only one expression of the phenomenon referred to as "localization" but it is certainly the most commonly encountered in Geomechanics. Localization in soils has been extensively studied experimentally, theoretically and numerically (e.g. McManus and Davis, 1997; Mokni and Desrues, 1999; Viggiani et al., 2001; Schrefler et al., 1996; Rice, 1976; Loret and Prevost, 1991; Runesson et al., 1996; Larsson et al., 1996; Liu et al., 2005; Kotronis et al., 2008, to name a few).

It is well known that when dealing with strain localization, the classical continuum mechanics does not apply anymore (Pijaudier-Cabot and Bažant, 1987). Although classical continuum mechanics can provide reasonable insight into the conditions under which strain localization may occur (Rice, 1976), such approach fails to predict the size of the shear band and the post-localization behavior.

One option to overcome this issue is to resort to an enhanced continuum approach where information about an internal length related to the width of the band is given. Several enhanced continuum techniques have been proposed in the literature and the reader can refer to Chambon et al. (2004) for a discussion about these. The local second gradient is one of these enhanced medium models that presents the advantage to have the kinematic enrichment independent from the constitutive equations used to describe the soil behavior. The local second gradient model is based on the pioneering work of Mindlin (1964, 1965) and Germain (1973a,b) and it has been derived in Chambon et al. (2001) for a monophasic material and extended to biphasic materials by Collin et al. (2006). Not only the local second gradient model can handle the post localization behavior but it was also demonstrated that the localization is mesh independent (Matsushima et al., 2002; Collin et al., 2006; Sieffert et al., 2009).

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As far as the authors are aware, no attempt has been made to apply an enhanced continuum approach to dilatant saturated material under deviatoric stress and undrained conditions. Collin et al. (2006) did not ascribe any dilation angle to the material they model so that the material remained biphasic (solid and water) throughout the simulation. Indeed, tests from the literature (McManus and Davis, 1997; Mokni and Desrues, 1999) show that, under the conditions above described, the pore pressure progressively shifts from the positive to negative range when the sheared specimen experiences dilation with formation of a third phase (gas) at cavitation. This modifies the effective stress state in a first instance, but also creates a perturbation in the system when cavitation occurs.

Not accounting for cavitation when modelling undrained testing of a saturated sample leads to a continuously increase of the global reaction force caused by the increase of effective stresses. This issue was shown to be particularly problematic with constitutive equations accounting for the degradation of the strength of materials (Loret and Prevost, 1991).

Using a dynamic numerical framework, Schrefler et al. (1996) modeled cavitation for a soil with high permeability (0.25×10^{-3} m/s) via a curve similar to a soil retention curve. However, the authors did not explicitly quantify the pressure at which cavitation is triggered. Also, the pore pressure post-cavitation follows the retention curve and is not limited to the partial vacuum created in the air phase. This is why unrealistic values of suction, as large as -8000 kPa, have been reached.

More recently, Liu and Scarpas (2005) conducted some numerical investigations on localization with hydro-mechanical coupling. They modeled a sandy type material with a range of permeability and observed variations in pore pressure that they attributed to cavitation. However, as far as we are aware, the cavitation process was not explicitly accounted for in their model and their conclusions seem questionable. In fact, the perturbation in pore pressure occurred at about -20 kPa, which is far from the physical cavitation pressure. The same year, Liu et al. (2005) looked at the effect of the material permeability on the formation of the bands. However, although the observations were relevant, the authors did not provide any in depth explanations of the results.

The aim of this paper is to provide some better understanding of the hydro-mechanical coupling in a saturated material subjected to deviatoric stress in undrained conditions. First, the biphasic second gradient model developed by Collin et al. (2006) has been extended to account for three phases. This is a necessary step to observe cavitation and highlight its effects on the mechanical response. The presentation of the new model, with the implementation of cavitation, constitutes the first part of this paper. Then, some insight is provided into the stability of shear bands with an emphasis on the effect of permeability, loading rate and dilation angle. As a result, a mapping of stability conditions pertaining to localized solutions is proposed. Finally, the effects of cavitation are discussed in the context of homogeneous and localized responses of specimens.

2. Formulation of the triphasic model

2.1. Local multiphasic second gradient

Phase changes are herein considered within a saturated specimen subjected to negative pore pressure. In other words, cavitation has been implemented. It corresponds to the transformation of liquid water into vapor water under reduction of pressure and at constant temperature. Note that the specimen is initially saturated; hence, there is no pre-existing air as such (occluded air nuclei are not modeled).

For the sake of clarity, some elements of terminology have to be defined before detailing the model: in the rest of the paper, "water" refers to liquid water occupying the pores of the specimen while "gas" refers to the vapor water having formed as a result of phase change.

Geomechanics convention is used: compressive stresses are positive.

The model presented in the following pertains to quasi-static conditions with large strains and in unsaturated conditions under Richard's assumptions (vapor water pressure is constant). In addition, isothermal conditions are assumed, grains are incompressible and undrained boundary conditions prevail: the total mass of the specimen is kept constant during the test.

The unknowns of the second gradient model are the (macro) displacement u_i and the microkinematic gradient v_{ij} while the variable pertaining to the liquid water flow equation is the pore water pressure p^w (possibly negative in unsaturated case).

The local second gradient is based on the assumption that the microkinematic gradient v_{ij} is equal to the macro-displacement gradient F_{ij}. This implies similar relations for virtual entities:

$$v_{ij} = \frac{\partial u_i}{\partial x_j} = F_{ij} \text{ and } v^*_{ij} = \frac{\partial u^*_i}{\partial x_j} = F^*_{ij} \tag{1}$$

Inspired by Terzaghi's formulation, an effective stress is herein defined as:

$$\sigma^t_{ij} = \sigma'^t_{ij} + S^{w,t}_r p^{w,t}\delta_{ij} + S^{g,t}_r p^g \delta_{ij} \tag{2}$$

where σ^t_{ij} is the total stress, σ'^t_{ij} is the effective stress, $p^{w,t}$ is the fluid pressure, p^g is the gas pressure, $S^{w,t}_r$ is the relative degree of saturation of water and δ_{ij} is Kronecker's delta. $S^{g,t}_r$ is the relative degree of saturation of gas defined as:

$$S^{w,t}_r + S^{g,t}_r = 1 \tag{3}$$

The density of the mixture is:

$$\varrho^{mix,t} = \varrho^s(1-\phi^t) + S^{w,t}_r \varrho^{w,t}\phi^t + (1-S^{w,t}_r)\varrho^g \phi^t \tag{4}$$

ρ^s is the density of the solid grains (assumed to be incompressible, i.e. ϱ^s = constant), $\varrho^{w,t}$ is the water density, $\emptyset^t$ is the porosity defined as $\phi^t = \Omega^{p,t}/\Omega^t$ where Ω^t is the current volume of a given mass of skeleton and $\Omega^{p,t}$ the corresponding porous volume.

In a weak form (virtual work principle), the balance equation of momentum for the mixture can thus be written as:

$$\int_{\Omega^t}\left(\sigma^t_{ij}\frac{\partial u^*_i}{\partial x^t_j} + \Sigma^t_{ijk}\frac{\partial^2 u^*_i}{\partial x^t_j \partial x^t_k}\right)d\Omega^t = \int_{\Omega^t}\varrho^{mix,t} g_i u^*_i d\Omega^t + \int_{\Gamma^t_\Omega}(\bar{t}_i u^*_i + \overline{T}_i Du^*_i)d\Gamma^t \tag{5}$$

where u^*_i is any kinematically admissible virtual displacement field, σ^t_{ij} is the Cauchy stress (total stress), Σ^t_{ijk} is the double stress dual of the virtual second micro kinematic gradient, x_i is the current coordinate, g_i is the gravity acceleration, $\bar{t}_i$ is the external (classical) forces per unit area and $\overline{T}_i$ an additional external (double) force per unit area, both applied on a part Γ^t_Ω of the boundary of Ω^t. Dq refers to the normal derivative of any quantity q (for instance $Du_i = n_k \partial u_i/\partial x_k$ where n_k is the normal to the assumed C1 boundary).

Eq. (5) above shows the coupling between stresses and pore pressure through the total stress. However, consistent with Collin et al. (2006), the water and gas pressures do not have any direct influence at microstructural level.

Note that the second gradient term is only activated after formation of a shear band and hence, it does not influence the plastic yielding of the specimen. The coupling between macro and micro scales is used to control the width of the plastic loading band.

In order to used C^0 functions for the displacement field (i.e. only first derivatives of the displacement), λ_{ij} Lagrange multipliers are