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Numerical analysis of quasi-static crack branching in brittle solids by a modified displacement discontinuity method

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ABSTRACT

Mechanism of quasi-static crack branching in brittle solids has been analyzed by a modified displacement discontinuity method. It has been assumed that the pre-existing cracks in brittle solids may propagate at the crack tips due to the initiation and propagation of the kink (or wing) cracks. The originated wing cracks will act as new cracks and can be further propagated from their tips according to the linear elastic fracture mechanics (LEFM) theory. The kink displacement discontinuity formulations (considering the linear and quadratic interpolation functions) are specially developed to calculate the displacement discontinuities for the left and right sides of a kink point so that the first and second mode kink stress intensity factors can be estimated. The crack tips are also treated by boundary displacement collocation technique considering the singularity variation of the displacements and stresses near the crack tip. The propagating direction of the secondary cracks can be predicted by using the maximum tangential stress criterion. An iterative algorithm is used to predict the crack propagating path assuming an incremental increase of the crack length in the predicted direction (straight and curved cracks have been treated). The same approach has been used for estimating the crack propagating direction and path of the original and wing cracks considering the special crack tip elements. Some example problems are numerically solved assuming quasi-static conditions. These results are compared with the corresponding experimental and numerical results given in the literature. This comparison validates the accuracy and applicability of the proposed method.

1. Introduction

The crack branching mechanism is a complicated process and has been usually treated dynamically. It is observed that as cracks travel faster in brittle solids such as glass, rocks and rock-like materials, they tend to branch out (Meyers, 1994). In the fracture mechanics literature, crack branching (bifurcation) is usually considered as a dynamic phenomenon and has been studied by many researchers (Yoffe, 1951; Ravi-Chandar and Knauss, 1984; Ravi-Chandar, 2004). Ravi-Chandar and Knauss (1984) made a critical analysis of crack branching and concluded that branching occurs at velocities much lower than those predicted by Yoffe (1951). They proposed a mechanism based on the formation of micro-cracks ahead of the main crack. The interactions of the micro-cracks among themselves and with the main crack dictate the branching response. In the present paper, it is concluded that the crack branching may occur even quasi-statically due to formation of wing and secondary cracks emanating from the crack tips and/or from the kink points (the points between the main crack and the propagating wing (or kink) cracks). After each incremental crack extension, the crack propagation angle will change and some kink points are produced where the stresses may exceed the strength of the fractured solid (due to the stress concentration phenomena at kink points) and the crack may start to branch out.

Several analytical, numerical and experimental works have been done on the subject (Sukumar et al., 1997; Hori et al., 2005; Golshani et al., 2005; Oliver et al., 2006; Rabczuk et al., 2007; Bordas et al., 2008; Stan, 2008; Oguni et al., 2009; Maysavina and Sadowski, 2009; Chen et al., 2012). In most of these studies, the importance of secondary cracks (which are recognized as shear cracks) has been emphasized. Secondary cracks are mainly observed in the experimental works and on the rock samples under compression (Hoek and Bieniawski, 1965; Bobet and Einstein, 1998; Sagong and Bobet, 2002;). In the author's recent works, it has been shown that the secondary cracks may be produced under both tensile and compressive loading conditions at the kinked points which may be considered as the possible points of high stress concentrations (Fatehi Marji et al., 2010; Fatehi Marji et al., 2011; Manouchehrian and Fatehi Marji, 2012; Fatehi Marji et al., 2012).

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Recently, the extended finite element method (XFEM) and the extended boundary element method (XBEM) have been developed for more sophisticated crack analysis. Mousavi et al. (2010) developed the harmonic enrichment functions which readily permit the XFEM to handle multiple interacting and branched cracks without any special treatment around the junction points. Bird et al. (2010) introduced the coupled boundary element–scaled boundary finite element method. Verification of the method is undertaken by means of estimating stress intensity factors and comparing them against analytical solutions. The coupled algorithm shows good convergence properties. Rabczuk et al. (2010) reviewed different crack tracking techniques in three-dimensions in detail and implemented them in the context of the extended element-free Galerkin method (XEFG). Garzon et al. (2011) modified the generalized finite element method (GFEM) which enables accurate modeling of problems involving multiple scales of interest using meshes with elements that are orders of magnitude larger than those required by the FEM.

A novel enriched Boundary Element Method (BEM) and Dual Boundary Element Method (DBEM) approach for accurate evaluation of stress intensity factors (SIFs) in crack problems was introduced by Simpson and Trevelyan (2011). They proposed an efficient numerical quadrature method for the evaluation of strongly singular and hypersingular enriched boundary integrals. Chen et al. (2012) used a strain smoothing procedure for the extended finite element method (XFEM) known as "edge-based" smoothed extended finite element method (ESm-XFEM). This method is tailored to linear elastic fracture mechanics (LEFM) and then the displacement-based approximation is enriched by the Heaviside and asymptotic crack tip functions using the framework of partition of unity. This eliminates the need for the mesh alignment with the crack and re-meshing, as the crack evolves and propagates.

More recently, the concept of isogeometric analysis describing CAD geometry are also used to approximate the unknown fields in a numerical discretisation by generating a boundary mesh representing a significant step in reducing the gap between engineering design and analysis (Simpson et al., 2013). Scott et al. (2013) proposed an isogeometric boundary element method (BEM), known as IGABEM, applied to two-dimensional elastostatic problems using non-uniform rational B-Splines (NURBS). They found that it is a natural fit with the isogeometric concept since both the NURBS approximation and BEM deal with quantities entirely on the boundary.

However, all of these methods give sophisticated and modern concepts of using analytical methods in combination with the finite and boundary element methods for estimating the accurate stress intensity factors and per sue the crack propagation process in cracked brittle solids based on the linear elastic fracture mechanics (LEFM). It should be noted that in all of these papers, it has been tried to incorporate the most accurate analytical approaches like the transformation and integration procedures in the numerical methods in order to reduce the meshing and/or eliminate the re-meshing in a crack propagation process. Displacement discontinuity method (which is a kind of indirect dual boundary element method) is vastly used for the analysis of the cracks by many researches (Guo et al., 1990; Shou and Crouch, 1995; Fatehi Marji et al., 2007; Fatehi Marji et al., 2010; Fatehi Marji et al., 2011; Haeri et al., 2013; Fatehi Marji, 2013). In this method, the two overlapped crack surfaces (in form of a line crack for two dimensional problems) are discretized simultaneously (unlike the direct dual boundary integral method where the two crack surfaces are discretized separately (Portela et al., 1992; Chen and Hong, 1999; Benedetti et al., 2008)). Displacement discontinuity approach reduces the meshing, incorporates the analytical solutions and gives very accurate results because the integrations and their derivatives on all of the boundary elements (i.e., for ordinary, kink and crack tip elements) are evaluated analytically (in a closed form). Therefore, in this paper, a semi-analytical higher order displacement discontinuity method is presented which uses the higher order displacement discontinuity formulations near the kink points. The cracks are traversing and discretizing in counterclockwise directions. Each kink point is considered as two overlapped crack tips and the mixed mode stress intensity factors near the kink (for both left and right sides of the kink) are computed.

2. Statement of the problem

Crack branching (bifurcation) is usually considered as a dynamic process. In the present paper, a quasi-static analysis of the crack branching mechanism is proposed by using linear elastic fracture mechanics (LEFM) concepts of stress intensity factors (SIFs). Based on these concepts, it has been concluded that the secondary cracks may start their propagation and causing crack branching phenomenon at the kink points. A two dimensional higher order kink element displacement discontinuity method is developed using the special kink elements for the treatment of the kinked and curved points of the crack and the special crack tip elements for the treatment of crack tips. This method estimates the kink stress intensity factors (Mode I and Mode II) at each side of the kinked points and at the tips of the wing and secondary cracks, respectively. Based on this approach, the left and right sides of a propagated kinked crack (consists of a main crack and a wing crack connecting at a kinked point) are considered as two adjacent crack tips (i.e., one tip on the left side and one tip on the right side of the kink element), then the relevant kink displacement discontinuities are evaluated using linear or quadratic displacement interpolation functions (specially developed for the kink point treatment).

The linear interpolation displacement function divides each kink element into two equal sub-elements one on the left side and another one on the right side of the kinked point. As the displacement discontinuities are estimated for the centers (nodes) of these sub-elements, the kinks stress intensity factors (Mode I and Mode II) can be estimated for one half of the sub element length and for both sides of the kinked point. In the literature of FEM and BEM, it has been numerically observed that the nodes situated at a distance equal to one quarter of the crack tip element length (measured from the crack tip and known as quarter point element or quarter point sub-element when using higher order elements) give a better estimation of the Mode I and Mode II stress intensity factors (Banerjee, 1994; Zienkiewicz and Taylor, 2000; Ameen, 2001; Sanford, 2003). The same achievement can be gained by the proposed method when using a quadratic kink element with two equal sub-elements on the left side and two equal sub-elements on the right side of the kinked point in a typical higher order kink element. It should be noted that the cubic variation of displacement discontinuities on the left and right sides of a kinked point in a kink element may also be used for estimating the Mode I and Mode II kink stress intensity factors. In this case, using equal sub-elements in each sides of the kink displacement discontinuity element, the left and right kink displacement discontinuities are situated at a distance equal to one twelfth of the total kink element length (Fatehi Marji and Dehghani, 2010). In this research, linear and quadratic kink elements are implemented in the boundary element code, the higher order cubic elements are used for treating the ordinary elements and the special crack tip elements are used for treating the crack tips (see Appendices A and B).

The crack propagation process can be investigated by implementing any mixed mode fracture criterion in the developed boundary element code. Based on linear elastic fracture mechanics (LEFM) theory, there are mainly three classic fracture criterion