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Discontinuous bifurcation analysis of thermodynamically consistent gradient poroplastic materials

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ABSTRACT

In this work, analytical and numerical solutions of the condition for discontinuous bifurcation of thermodynamically consistent gradient-based poroplastic materials are obtained and evaluated. The main aim is the analysis of the potentials for localized failure modes in the form of discontinuous bifurcation in partially saturated gradient-based poroplastic materials as well as the dependence of these potentials on the current hydraulic and stress conditions. Also the main differences with the localization conditions of the related local theory for poroplastic materials are evaluated to perfectly understand the regularization capabilities of the non-local gradient-based one. Firstly, the condition for discontinuous bifurcation is formulated from wave propagation analyses in poroplastic media. The material formulation employed in this work for the spectral properties evaluation of the discontinuous bifurcation condition is the thermodynamically consistent, gradient-based modified Cam Clay model for partially saturated porous media previously proposed by the authors. The main and novel feature of this constitutive theory is the inclusion of a gradient internal length of the porous phase which, together with the characteristic length of the solid skeleton, comprehensively defined the non-local characteristics of the represented porous material. After presenting the fundamental equations of the thermodynamically consistent gradient based poroplastic constitutive model, the analytical expressions of the critical hardening/softening modulus for discontinuous bifurcation under both drained and undrained conditions are obtained. As a particular case, the related local constitutive model is also evaluated from the discontinuous bifurcation condition stand point. Then, the localization analysis of the thermodynamically consistent non-local and local poroplastic Cam Clay theories is performed. The results demonstrate, on the one hand and related to the local poroplastic material, the decisive role of the pore pressure and of the volumetric non-associativity degree on the location of the transition point between ductile and brittle failure regimes in the stress space. On the other hand, the results demonstrate as well the regularization capabilities of the non-local gradient-based poroplastic theory, with exception of a particular stress condition which is also evaluated in this work. Finally, it is also shown that, due to dependence of the characteristic lengths for the pore and skeleton phases on the hydraulic and stress conditions, the non-local theory is able to reproduce the strong reduction of failure diffusion that takes place under both, low confinement and low pore pressure of partially saturated porous materials, without loosing, however, the ellipticity of the related differential equations.

1. Introduction

Quasi-brittle materials like soils and concrete have very complex mechanical behaviors when subjected to load histories involving large accumulated inelastic deformations. In these cases, and from the analytical stand point, the evaluation and prediction of the involved failure mode and the location of the transition point between brittle and ductile failure regimes becomes very complex. This is mostly due to the diversity of governing parameters and the variability of their roles in the mechanical degradation processes of these complex materials.

Regarding non-porous continua-based material theories, many authors performed studies to evaluate the post peak behavior and, moreover, the discontinuous bifurcation potentials through the analytical determination of the related critical hardening modulus (see a.o. Zhang et al., 2005; Ottosen and Runesson, 1991;

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Runesson et al., 1991, 1996; Perić and Rasheed, 2007). The evaluations covered not only classical local material theories but also anisotropic material formulations (Zhang et al., 2005), as well as constitutive theories related to fiber reinforced materials (Etse et al., 2012; Perić and Rasheed, 2007) and, moreover, to enhanced non-local, gradient-based, materials (Vrech and Etse, 2012).

In case of quasi-brittle porous materials, and due to the diversity and complexity of the involved variables during inelastic degradation process, the discontinuous bifurcation condition was analyzed, so far, by means of different approaches and taking into account the influence of the Lode angle (Zhen et al., 2010), water content or fluid pressure (Liu and Scarpas, 2005), porosity (Zhang et al., 2002), permeability (Zhang and Schrefler, 2001), temperature (Sulem, 2010), etc. Actually, most of the published localization analysis in porous materials (see a.o. Sabatini and Finno, 1996; Benallal and Comi, 2002; Borja, 2004; Ehlers et al., 2004; Kristensson and Ahadi, 2005; Schiava and Etse, 2006), are based on the restrictive consideration that discontinuous bifurcations may only occur in the solid phase. Therefore, and despite the numerous proposals, there is still a need of accurate evaluations of the potentials for discontinuous bifurcation in partially saturated porous materials under more general or arbitrary conditions. This is particularly the case when it comes to assessing the influence of hydraulic and mechanical states of the porous and solid phases, respectively, in the failure modes of partially saturated porous materials, whether brittle or ductile.

Recently, Mroginski et al. (2011), proposed a thermodynamically consistent gradient-based constitutive theory for partially saturated quasi-brittle porous material. It follows the thermodynamic gradient-based formulations for continuous (non-porous) materials by Svedberg and Runesson (1997) and Vrech and Etse (2009), whereby the state variables are the only ones of non-local characters.

The proposal by Mroginski et al. (2011), introduces a novel aspect to more effectively capture the strong influence in failure behaviors of partially saturated porous materials of both the hydraulic and mechanical states of their microstructure. It considers two independent characteristic lengths, for the porous and solid phases, respectively, which are defined in terms of the governed water content and confining pressure. This is in line with the philosophy by Schrefler et al. (2006), in the sense that multiple internal lengths shall be considered to realistically reproduce the strong variation of failure modes of porous material. The capabilities of the thermodynamically consistent gradient-based constitutive theory to predict the variation from localized (brittle) to distributed (ductile) failure modes of quasi-brittle porous materials like soils under different conditions of the water content and confining pressure are demonstrated in Mroginski and Etse (2013).

In this work the discontinuous bifurcation conditions for partially saturated porous materials like soils under both, drained and undrained hydraulic conditions, are numerically and analytically evaluated. The modified Cam Clay model is considered in the framework of the thermodynamically consistent gradient-based formulation by Mroginski et al. (2011).

The numerical study of discontinuous bifurcation condition is based on the identification of the stress domain (under drained and undrained conditions) where the singularity of the localization tensor is fulfilled. On the other hand, the analytical procedure requires the explicit formulation of the critical (minimum) hardening modulus corresponding to the first condition for discontinuous bifurcation in the deformation history of partially saturated porous materials. This is done by means of an extension to the cases of local and gradient-based porous continua of the analytical solutions for discontinuous bifurcation by Ottosen and Runesson (1991), Runesson et al. (1991) and Perić and Rasheed (2007).

After summarizing the most relevant equations of the constitutive theory by Mroginski et al. (2011), this work focuses on formulating its localization tensor and, further, the explicit solutions of the critical hardening modulus for discontinuous bifurcation under both, drained and undrained conditions. The particular cases of plane stress and plain strain are considered.

Then, the potentials for discontinuous bifurcation by means of both the numerical and analytical methods are evaluated for all possible stress states along the first yield surface and the critical state line of both the local and gradient-based Cam-clay constitutive theories for partially saturated poroplastic materials. Drained and undrained hydraulic conditions under both plane strain and plane stress are considered in the analyses.

Regarding the local Cam-clay model for partially saturated porous material the results demonstrates that the position of the transition point for brittle–ductile failure modes in the stress space, and the overall failure mode, do strongly depend on the particular hydraulic and confinement conditions. In this sense, drained hydraulic conditions are more critical for discontinuous bifurcations as well as plane stress conditions.

In case of the gradient-based poroplastic model, the results indicate that its regularization capabilities are able to suppress the discontinuous bifurcation conditions of the local model for all possible stress state, with exception of some particular cases that are evaluated in this work. Nevertheless, and due to the particular form of the gradient characteristic lengths, the non-local model reproduces the increasing degradation of the localization tensor spectral properties as the stress state goes into the small confinement regime and, also, as the pore pressure reduces. In these regimes the non-local model leads to strong reductions of the failure diffusion or quasi-brittle failure modes (minimum eigenvalue of the localization tensor very close to zero). This spectral properties degradation is more critical under drained conditions. Theses results confirm the capabilities of the non-local formulation in this work to reproduce the influence of the stress and hydraulic conditions in the failure modes, in accordance with the experimental evidence on porous media (Vardoulakis, 1996; Sawicki and Świdziński, 2010; Cetin and Gökoĝlu, 2013) and theoretical developments (Runesson et al., 1996; Zhang and Schrefler, 2001; Al Hattamleh et al., 2004).

2. Thermodynamically consistent gradient-based constitutive theory for non-saturated porous media

Porous media are multiphase systems with interstitial voids in the grain matrix filled with water (liquid phase), water vapor and dry air (gas phase) at microscopic level. The mechanical behavior of partially saturated porous media is usually described by the effective stress tensor σ'_{ij}, as follows

$$\sigma'_{ij} = \sigma_{ij} - \delta_{ij} p^w = \sigma^n_{ij} + s_{ij} \tag{1}$$

being σ_{ij} the total stress and $s_{ij} = \delta_{ij}(p^a - p^w)$ and $\sigma^n_{ij} = \sigma_{ij} - \delta_{ij} p^a$ the net and suction stress tensors, respectively, while δ_{ij} is the Kronecker delta. Thereby are p^a and p^w the gas and water pore pressures, respectively. In several geotechnical problems the gas pore pressure can be considered as a constant term that equals the atmospheric pressure. In these cases the suction tensor is counterpart to the water pore pressure, p.

Plastic behavior of quasi-brittle porous materials is related to permanent skeleton strains, but also to permanent variations in fluid mass content, m, due to related porosity variations. To fully characterize current stages of poroplastic media and to describe their inelastic evolutions, the plastic porosity or plastic fluid mass content m^p must be considered in addition to the plastic strain ε^p_{ij},