International Journal of Solids and Structures 51 (2014) 1847–1858

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Stretch-induced wrinkling of polyethylene thin sheets: Experiments and modeling

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ARTICLE INFO

Article history:
Received 29 October 2013
Received in revised form 6 January 2014
Available online 6 February 2014

Keywords:
Thin sheet
Buckling
Wrinkle
Viscoelasticity

ABSTRACT

This paper presents a study on stretch-induced wrinkling of thin polyethylene sheets when subjected to uniaxial stretch with two clamped ends. Three-dimensional digital image correlation was used to measure the wrinkling deformation. It was observed that the wrinkle amplitude increased as the nominal strain increased up to around 10%, but then decreased at larger strain levels. This behavior is consistent with results of finite element simulations for a hyperelastic thin sheet reported previously (Nayyar et al., 2011). However, wrinkles in the polyethylene sheet were not fully flattened out at large strains (>30%) as predicted for the hyperelastic sheet, but exhibited a residual wrinkle whose amplitude depended on the loading rate. This is attributed to the viscoelastic response of the material. Two different viscoelastic models were adopted in finite element simulations to study the effects of viscoelasticity on wrinkling and to improve the agreement with the experiments, including residual wrinkles and rate dependence. It is found that a parallel network model of nonlinear viscoelasticity is suitable for simulating the constitutive behavior and stretch-induced wrinkling of the polyethylene sheets.

1. Introduction

In a previous paper (Nayyar et al., 2011), we studied wrinkling of a rectangular hyperelastic thin sheet subjected to uniaxial stretch with two clamped ends (see Fig. 1) through finite element simulations. It was found that the formation of stretch-induced wrinkles depends on the applied nominal strain and two geometrical ratios, the length-to-width aspect ratio ($\alpha = L_0/W_0$) and the width-to-thickness ratio ($\beta = W_0/h_0$). The wrinkle wavelength was shown to decrease with increasing strain, in good agreement with a previous prediction using a scaling analysis (Cerda et al., 2002; Cerda and Mahadevan, 2003). However, the wrinkle amplitude was found to increase with strain until $\sim$10% and then decrease, eventually flattening completely beyond a moderately large strain ($\sim$30%); in contrast, the scaling analysis predicted monotonically increasing wrinkle amplitude. Similar wrinkling problems have been studied by others, mostly using analytical or numerical methods (Segedin et al., 1988; Friedl et al., 2000; Jacques and Potier-Ferry, 2005; Zheng, 2009; Puntel et al., 2011; Kim et al., 2012; Healey et al., 2013). Very few experimental results have been reported (Cerda et al., 2002; Zheng, 2009). The experimental data in Cerda et al. (2002) showed that the measured wrinkle wavelengths in a polyethylene sheet agreed well with the scaling analysis, but no data for the wrinkle amplitude was reported. Zheng (2009) measured stretch-induced wrinkle profiles in silicone membranes using an optical fringe projection method and found that the wrinkle amplitude decreased with increasing strain. The measurement however did not show increasing wrinkle amplitude at the early stage of stretching. In this paper, we present a detailed experimental study on stretch-induced wrinkling and then compare the results with numerical simulations assuming nonlinear elastic and viscoelastic material models. We note that experimental measurements of wrinkles have been reported by others for thin sheets subjected to different loading conditions such as shear and corner loadings (Jenkins et al., 1998; Blandino et al., 2002; Wong and Pellegrino, 2006).

The remainder of this paper is organized as follows. Section 2 describes the experimental procedures and techniques used to generate and measure wrinkle profiles. Section 3 presents the experimental results. In Section 4, modeling and simulations of stretch-induced wrinkling with different material models are presented, emphasizing the effects of viscoelasticity. Section 5 discusses the comparison between experiments and modeling, followed by a brief summary in Section 6.

2. Experimental method

The material used in this study was a commercial grade polyethylene (Husky black plastic sheeting, manufactured by

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http://dx.doi.org/10.1016/j.ijsolstr.2014.01.028
0020-7683/

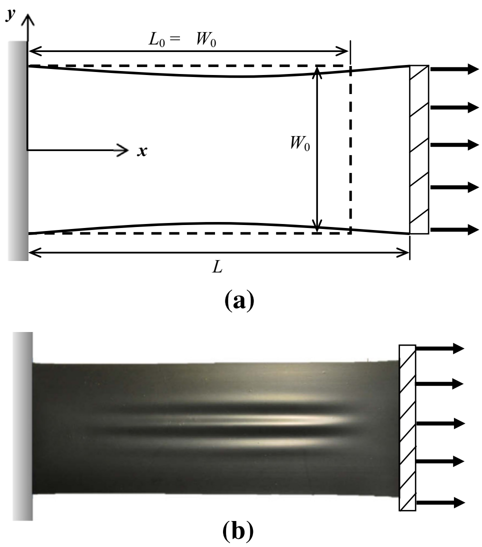

Fig. 1. (a) Schematic illustration of a rectangular sheet with two clamped-ends, subject to uniaxial stretch. (b) An optical image of a stretch-wrinkled polyethylene sheet ($\varepsilon \sim 10\%$).

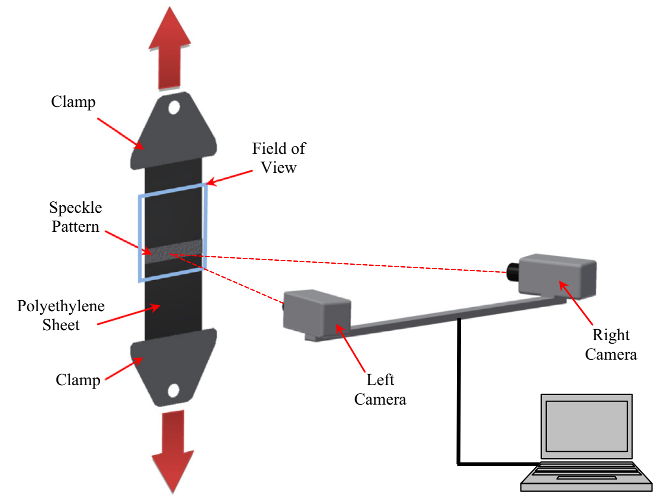

Fig. 2. Schematic illustration of the 3D-DIC setup used for stretching tests and wrinkle measurements with a clamped thin sheet.

Poly-America) in the form of a thin sheet with nominal thickness, $h_0 = 0.1$ mm. The first set of specimens had in-plane dimensions (after clamping) of: $L_0 = 250$ mm and $W_0 = 100$ mm, with the length-to-width aspect ratio $\alpha = 2.5$ and the width-to-thickness ratio $\beta = 1000$. A second set of specimens was prepared with $L_0 = 200$ mm and hence $\alpha = 2$. It was found that the as-received polyethylene sheet was not perfectly flat but had initial wrinkles and creases. In order to make the sheet flat, each specimen was first heat-treated by placing the sheet in between two plexiglas plates at a temperature of 75 °C for 24 h, and then cooling slowly to room temperature. This process helped in removing the creases and attaining a flat sheet with negligible initial curvature.

Next, a speckle pattern was made on the surface of the specimen by using a white gel pen of 0.7 mm tip radius, covering the whole width of the specimen, with a height of approximately 15 mm in the middle of the specimen. This speckle pattern was used to determine the displacements and strains using the three-dimensional digital image correlation (3D-DIC) technique. The 3D-DIC technique is a combination of the stereo-vision technique and the digital image correlation (DIC) (Luo et al., 1993; Sutton et al., 2009). It relates the 3D geometry to a reference state (before deformation) so that the corresponding displacements and displacement gradients can be determined. Several references on applications of 3D-DIC in experimental mechanics can be found in Orteu (2009). This technique was used in the present study to measure three-dimensional geometry of the wrinkles in polyethylene sheets.

After forming the speckle pattern, the specimen was clamped at the two ends in a specially designed jig (Nayyar, 2013), which allowed proper alignment of the sheet with respect to the clamps. To hold the clamped specimen in the Instron tension test machine, a pinned support was used at each end so that the clamps were free to rotate with respect to the pin, minimizing the twisting moment due to any misalignment. After installing the clamped polyethylene sheet specimen onto the Instron machine, a small tensile strain ($\leqslant 0.5\%$) was applied to reduce the initial undulations and slack in the sheet. The polyethylene sheet was then stretched up to 140% under displacement control at two different strain rates: 0.0169 s^{-1} and 0.00169 s^{-1}. The evolution of the wrinkle amplitude with time was recorded using the 3D-DIC system. Fig. 2 shows the experimental setup: two CCD cameras, each with a resolution of 1624 $\times$ 1236 pixels were used to record the central portion of the specimen that was decorated with the speckle pattern. The two cameras were placed approximately at a distance of 1.3 m away from the specimen with a distance of 0.45 m from each other. This scheme along with a 50 mm lens resulted in a field of view of about 100 mm $\times$ 100 mm. The field of view of the two cameras was adjusted in such a way that the speckled region of the specimen was always visible as the sheet was stretched up to 140%. A commercial 3D-DIC software package, ARAMIS, was used to register the images from the two cameras and to calculate the displacement and strain field over the speckled region. Using this setup, the polyethylene sheet specimen was found to have a resolution of 9–10 pixels/mm, providing a displacement resolution of $\sim$0.02 mm. A total of 19 specimens were measured as summarized in Table 1.

3. Experimental results

Fig. 3 shows the optical images and the wrinkle profiles of a polyethylene sheet ($\alpha = 2.5$) stretched to different nominal strain levels ($\varepsilon = 0, 0.1, 0.2, 0.3$ and 0.4) at a strain rate of $\dot{\varepsilon} = 0.00169$ s^{-1}. Referring to Fig. 1, the nominal strain is defined as, $\varepsilon = \delta/L_0$, where $\delta = L - L_0$ is the end displacement. The wrinkle profiles were obtained from the out-of-plane displacement, $u_z(L/2, y)$, measured by 3D-DIC along the line $x = L/2$. It should be noted that the sheet was not perfectly flat at zero strain even after the heat treatment. The observed initial undulations are partly due to small geometric misalignments at the clamped ends and partly due to gravity loading, which are eliminated once a small tensile strain is applied. Subsequently, stretch-induced wrinkles form near the center of the sheet, with a profile that appears

Table 1
Summary of wrinkle measurements.

Aspect ratio α	2.5	2.5	2	2
Strain rate (s^{-1})	0.00169	0.0169	0.00169	0.0169
Number of specimens	5	7	3	4
Average peak amplitude (mm)	0.368	0.343	0.538	0.523
Standard deviation (mm)	0.045	0.039	0.027	0.082
Average strain at peak amplitude	0.111	0.109	0.100	0.088
Standard deviation	0.017	0.022	0.004	0.007
Average amplitude at 30% strain (mm)	0.166	0.133	0.245	0.112
Standard deviation (mm)	0.030	0.024	0.035	0.076