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Effective elastic shear stiffness of a periodic fibrous composite with non-uniform imperfect contact between the matrix and the fibers

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In this contribution, effective elastic moduli are obtained by means of the asymptotic homogenization method, for oblique two-phase fibrous periodic composites with non-uniform imperfect contact conditions at the interface. This work is an extension of previous reported results, where only the perfect contact for elastic or piezoelectric composites under imperfect spring model was considered. The constituents of the composites exhibit transversely isotropic properties. A doubly periodic parallelogram array of cylindrical inclusions under longitudinal shear is considered. The behavior of the shear elastic coefficient for different geometry arrays related to the angle of the cell is studied. As validation of the present method, some numerical examples and comparisons with theoretical results verified that the present model is efficient for the analysis of composites with presence of imperfect interface and parallelogram cell. The effect of the non uniform imperfection on the shear effective property is observed. The present method can provide benchmark results for other numerical and approximate methods.

1. Introduction

The paper addresses the problem of computation of the effective elastic properties of heterogeneous materials with imperfect bonding between the matrix and the inhomogeneities. We address the case of long fiber reinforced composite (plane strain problem and anti-plane problem). There are two ways proposed in literature to model the mentioned imperfections: (A) continuum approach when the layered inhomogeneity is considered directly and (B) discrete model when the imperfection is modeled by springs of certain stiffnesses distributed along the interface.

To the best of our knowledge, first results on inhomogeneous interface has been obtained by Kanaun and Kudriavtseva (1983, 1986) where effective elastic properties are calculated for a material containing spherical and cylindrical inhomogeneities, correspondingly, surrounded by radially inhomogeneous interphase zones. In these papers, the basic idea of replacing an inhomogeneous inclusion by an equivalent homogeneous one was formulated. Such a replacement was carried out by modeling the inhomogeneous interface by a number of thin concentric layers (piecewise constant variation of properties). The basic idea of replacing inhomogeneous inclusions by equivalent homogeneous ones has been utilized in the majority of works on the topic.

The idea of approximating radially variable properties by multiple layers (piecewise constant variation of properties) was explored by Garboczi and Bentz (1997) and Garboczi and Berryman (2000) in the context of applications to concrete composites. An alternative method was used by Wang and Jasiuk (1998). They considered a general composite material with spherical inclusions representing the interphase as a functionally graded material and calculated effective elastic moduli using the composite spheres assemblage method for the effective bulk modulus and the generalized self-consistent method for the effective shear modulus. As far as an arbitrary law of radial variation in properties is concerned, apart from the above mentioned idea of multilayer approximation, an interesting methodology was proposed by Shen and Li (2003, 2005), whereby the thickness of the interface is increased in an incremental, "differential" manner, with homogenization at each step. This idea, with modifications, has been utilized by

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Sevostianov (2007) and Sevostianov and Kachanov (2007) for particles reinforced composites with interphase layers. The procedure of homogenization is reduced to solving non-linear ordinary differential equation.

Elastic composites with imperfect contact adherence have different area of applications. For instance, asphalt concrete (AC) is a typical multi-phase composite material. The characteristics of each constituent material in this composite and their interactions all contribute to the overall performance of the asphalt pavement, which may also be affected by the distribution and the volume fractions of these components. Particularly, these factors include the modulus and fractions of coarse aggregates and asphalt mastic which consists of asphalt and fine aggregates, air void fraction, and so on. The conventional continuum based models cannot take account of these factors in the analysis and design, and hence fail to quantitatively capture the complex mechanical behavior of the AC upon loading.

Other than the continuum approaches, micromechanical models can account for the roles of constituent materials playing in a composite. Many micromechanical-based models have been proposed to simulate the mechanical behavior of AC, for instance, see Zhu et al. (2011). However, few researchers have reported the work of the effect of the interfacial bonding strength between aggregate and asphalt mastic on the behavior of AC, which may have significant effect on the mechanical properties and failure mechanisms as well as the strengths of AC. For example, it is observed that the asphalt concrete can be easily debonded at the interface between aggregate and asphalt mastic by a fatigue loading. Craus et al. (1978) believed that the interfacial bonding strength was due to the physic-chemical reaction, which was related to the property of asphalt, geometry, size as well as surface activity of aggregate. Therefore, it is almost impossible to find a perfect interfacial bond existing between asphalt mastic and aggregates, and it may be inappropriate to describe the physical nature and macro-mechanical behavior of AC by considering the bond as the perfect kind.

An alternative approach has been, to the best of our knowledge, first proposed by Hashin (1990) who analized imperfect interface conditions in terms of linear relations between interface tractions and displacement jumps. All the thermoelastic properties of unidirectional fiber composites with such interface conditions are evaluated on the basis of a generalized self-consistent scheme model. Hashin (2002) reported that the imperfect interphase conditions are equivalent to the effect of a thin elastic interphase, and high accuracy of the method is proved by comparison of solutions of several problems in terms of the explicit presence of the interphase as a third phase. Hashin's approach was used for spherical particle-reinforced inhomogeneities by Benveniste and Miloh (2001) and Wang et al. (2005). Sevostianov et al. (2012) compared the two approaches for the case of incompressible layer between the phases and calculatee effective properties of fiber reinforced composites with periodic square arrays of fibers possessing imperfect contact with the surrounding material. They identified the interval of thickness at which the interphase does not influence the effective properties and show how the imperfection effects described by different models can be expressed in terms of each other. Guinovart-Díaz et al. (2013) used this approach for the case of a composite with parallelogram-like cell of periodicity using approach developed by Molkov and Pobedria (1985). The present work generalizes results of Guinovart-Díaz et al. (2013) to the case when the imperfectness of the contact between the matrix and the fibers is non-uniform, i.e. properties of the interface depend on both the angular and radial coordinates of a point. The aforementioned works differ to Andrianov et al. (2005, 2008) among others where homogenization method for evaluating effective properties and determining the micro-mechanical response is applied under ideal contact assumption at the interface.

2. Formulation of the problem

We consider a unidirectional periodic two-phase fiber reinforced composite shown in Fig. 1. All the fibers are assumed to be of circular cross sections with radius R. The material properties of each phase are transversely-isotropic with the axes of material and geometric symmetry being parallel. The angle of the cell θ is assumed to remain constant so that a parallelogram cell with periods w_1, w_2 can be defined. The periodicity of microstructure determines the geometry of the periodic cell S (Fig. 2). The contact between the matrix S_1 and the fibers S_2 is assumed to be non-perfect along the interface $\Gamma = \left\{z = Re^{i\theta}, \quad 0 \leq \theta \leq 2\pi\right\}$.

In order to model various possible damages occurring on the fiber–matrix interface composite the non uniform spring formulation of imperfect bonded are considered using a generalized shear lag model (Hashin, 1990, 1991a,b), which is also called the mechanically compliant interface: tractions are assumed to be continuous across the interface while displacements may be discontinuous. The jumps in displacement components are further assumed to be proportional, in terms of the "spring-factor-type" interface parameter, to their respective interface traction component $T_3 = \sigma_{\gamma 3} n_\gamma, \quad \gamma = 1,2$

$$T_3^{(1)} = T_3^{(2)} = \widehat{K}_s(R,\theta)\|u_3\|, \quad \text{on} \quad \Gamma. \tag{1}$$

This traction component T_3 is tangential to the interface and n is the normal unit vector to the interface Γ. $\widehat{K}_s$ is a function of the position at the interface which is called proportional interface parameter, and index "s" indicates the shear proportional spring factor. The double bar notation is used to denote the jump of the relevant function across the interphase Γ taken from the matrix (1) to the fiber (2) i.e. $\|f\| = f_1 - f_2$. The Eq. (1) is usually called a weak interface condition. It has been originally proposed by Goland and Reissner (1944) and used later in works of Benveniste and Miloh (2001), Molkov and Pobedria (1988), Mahiou and Beakou (1998), Andrianov et al. (2007).

3. Asymptotic homogenization method for the anti-plane problem

In a two-dimensional situation of uniaxially reinforced composite, the system of equations of elasticity separate in plane-strain and anti-plane-strain deformation states (see, for example, Pobedria, 1984). First of them involves in-plane displacements u_1 and u_2, while the other one, which is of particular interest in this work,

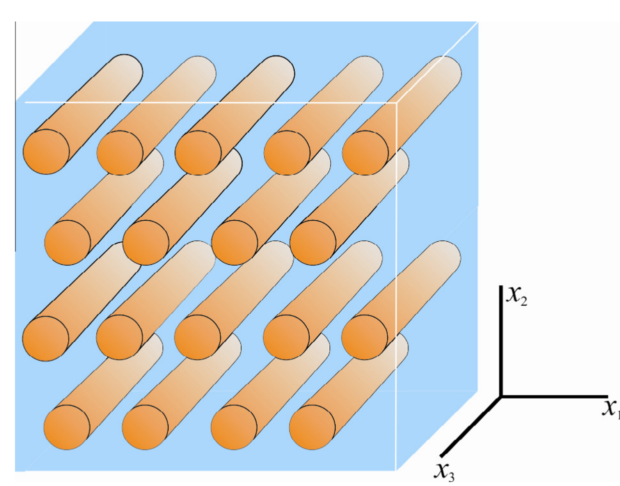

Fig. 1. The heterogeneous medium and extracted the rhombic periodic cell.