International Journal of Solids and Structures 51 (2014) 1340–1349

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Theoretical model of crack branching in magnetoelectric thermoelastic materials

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ARTICLE INFO

Article history:
Received 20 September 2013
Received in revised form 11 December 2013
Available online 21 December 2013

Keywords:
Thermomagnetoelectroelastic
Crack branching
Thermal stresses
Intensity factors

ABSTRACT

Thermomagnetoelectroelastic crack branching of magnetoelectro thermoelastic materials is theoretically investigated based on Stroh formalism and continuous distribution of dislocation approach. The crack face boundary condition is assumed to be fully thermally, electrically and magnetically impermeable. Explicit Green's functions for the interaction of a crack and a thermomagnetoelectroelastic dislocation (i.e., a thermal dislocation, a mechanical dislocation, an electric dipole and a magnetic dipole located at a same point) are presented. The problem is reduced to two sets of coupled singular integral equations with the thermal dislocation and magnetoelectroelastic dislocation densities along the branched crack line as the unknown variables. As a result, the formulations for the stress, electric displacement and magnetic induction intensity factors and energy release rate at the branched crack tip are expressed in terms of the dislocation density functions and the branch angle. Numerical results are presented to study the effect of applied thermal flux, electric field and magnetic field on the crack propagation path by using the maximum energy release rate criterion.

1. Introduction

Nowadays, materials with coupling magnetic, electric and mechanical effect have been found increasingly applications in modern industries such as multilayer actuators, sensors, controlling devices, smart and intelligent structures, and biological devices. These materials are often made in the form of composites with a piezoelectric inclusions and piezomagnetic matrix phase (Huang and Kuo, 1997; Kirchner and Alshits, 1996; Nan, 1994). Most materials with magnetoelectric coupling are ceramics. They can fail prematurely due to defects such as cracks arising in the manufacturing process when subjected to thermal, mechanical, electric and magnetic loads. There has been tremendous interest in studying the fracture and failure behaviors of magnetoelectric materials since the importance of the reliability of these devices. Green's functions for an infinite two-dimensional anisotropic magnetoelectroelastic medium containing an elliptical cavity were obtained by Liu et al. (2001). The magnetoelectroelastic problem of a crack in a medium possessing coupled piezoelectric, piezomagnetic and magnetoelectric effects was considered by Wang and Mai (2003). Green's functions for a defect in an infinite magnetoelectroelatic solid induced by the thermal analog of a line temperature discontinuity and a line heat source were derived in closed form by Qin (2005). The different electromagnetic boundary conditions on the crack-faces in magnetoelectroelastic materials were discussed by Wang and Mai (2007). The problem of collinear unequal crack series under mode I magneto-electro-mechanical loadings was studied by Li and Lee (2010).

On the other hand, the phenomenon of crack branching is an important aspect of piezoelectric, piezomagnetic and magnetoelectric materials fracture mechanics. The direction of crack branching can be one of the major factors in determining the residual strength of the structural components. Park and Sun (1995) reported that the crack propagation deviated from its original direction under the combined mechanical and electrical load in their three-point bending test with an unsymmetrical crack in a PZT-4 specimen. The problems of crack branching in a piezoelectric solid were investigated by continuous distribution of edge dislocation and electric dipole method by Zhu and Yang (1999), Xu and Rajapakse (2000). The problem of crack deflection in bimaterial systems with various materials combinations was solved by Qin and Zhang (2000). The effect of a transverse electric field on crack kinking in ferroelectric ceramics subjected to purely electrical load was investigated by Jeong et al. (2008). Tian and Rajapakse (2008) presented a theoretical model to determine the fracture parameters of a finite impermeable crack with one or more branches in a magnetoelectroelstic plane subjected to the remote mechanical, electrical and magnetic loading.

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http://dx.doi.org/10.1016/j.ijsolstr.2013.12.025

However, in contrast to the thermoelastic, thermopieozelectric and thermomagnetoelectroelastic straight crack problem, very few papers can be found on the thermoelastic, thermopiezoelectric and thermomagnetoelectroelastic crack branching problems due to the complicated coupling or interaction between thermal effects and magnetoelectroelastic loading. A thermoelastic problem for an infinite plate with a kinked crack was analyzed by Hasebe et al. (1986). The two-dimensional problem of curvilinear cracks lying along the interface between dissimilar materials under remote heat flux was considered by Chao and Shen (1993). Crack growth prediction of an inclined crack in a half-plane thermopiezoelectric solid was studied by Qin and Mai (1997). Solutions to the thermoelastic crack branching in general anisotropic media and the thermoelastic interface crack branching in dissimilar anisotropic bi-materials media were presented by Li and Kardomateas (2005, 2006). Zhang and Wang (2013) studied thermopiezoelectric crack branching of piezoelectric materials based on extended Stroh formalism (Stroh, 1958) and continuous distribution of dislocation approach. In the work of Qin and Mai (1997), the minimum strain energy density criterion was used, i.e., fracture initiates from an interior element located at a finite distance r_0 from the crack front. The direction of crack propagation is determined by the theory of maximum energy release rate criterion in our previous work (Zhang and Wang, 2013) and this paper, that is, the branching angle at which makes the energy release rate attain its maximum value.

The purpose of this paper is to present a theoretical model for the evaluation of fracture mechanics parameters of a branched crack in a thermomagnetoelectroelastic medium subjected to remote thermal, mechanical, electric and magnetic loading. The plan of the paper is as follows. In Section 2 we outline the basic theory of extended Stroh formalism. In Section 3 a closed form solution is obtained for the interaction between a crack and a thermomagnetoelectroelastic dislocation. In Sections 4 and 5, the branched portion of the crack is modeled by a continuous distribution of thermal dislocation and magnetoelectroelastic dislocation technique, leading to two sets of coupled singular integral equations in terms of unknown dislocation density functions. Some numerical results are presented in Section 6, and concluding remarks are made in Section 7.

2. The Stroh formalism

Consider a linear magnetoelectroelastic material in which all fields are assumed to depend only on the in-plane coordinates x_1 and x_2. The shorthand notation developed by Barnett and Lothe (1975) based upon Stroh formalism (Stroh, 1958) is adopted in this paper. Lower case Latin subscripts always range from 1 to 3, upper case Latin subscripts will range from 1 to 5, and the summation convention is used for repeating subscripts unless otherwise indicated. In the stationary case when no free electric charge, electric current, body force or heat source exists, the basic equations for thermomagnetoelectroelastic materials can be written as (Mindlin, 1974; Qin, 2005)

$$h_{i,i} = 0, \quad \mathbf{\Pi}_{iJ,i} = 0 \tag{1}$$

together with

$$h_i = -k_{ij}T_{,j}, \quad \mathbf{\Pi}_{iJ} = E_{iJKm}u_{k,m} - \chi_{iJ}T \tag{2}$$

in which

$$\mathbf{\Pi}_{iJ} = \begin{cases} \sigma_{ij}, J \leqslant 3 \\ D_i, \ J = 4 \\ B_i, \ J = 5 \end{cases} \tag{3}$$

$$u_K = \begin{cases} u_k, K \leqslant 3 \\ \phi, \ K = 4 \\ \varphi, \ K = 5 \end{cases} \tag{4}$$

$$\chi_{iJ} = \begin{cases} \beta_{ij}, K \leqslant 3 \\ \gamma_i, \ K = 4 \\ \upsilon_i, \ K = 5 \end{cases} \tag{5}$$

$$E_{iJKm} = \begin{cases} c_{ijkm}, & J, \ K \leqslant 3 \\ e_{mij}, & J \leqslant 3, \ K = 4 \\ h_{mij}, & J \leqslant 3, \ K = 5 \\ e_{ikm}, & J = 4, \ K \leqslant 3 \\ -\kappa_{im}, & J = 4, \ K = 4 \\ -\alpha_{im}, & J = 4, \ K = 5 \\ h_{ikm}, & J = 5, \ K \leqslant 3 \\ -\alpha_{im}, & J = 5, \ K = 4 \\ -\mu_{im}, & J = 5, \ K = 5 \end{cases} \tag{6}$$

where T and h_i are temperature change and thermal flux, σ_{ij}, D_i and B_i are elastic stress tensor, electric displacement vector and magnetic induction vector; u_i, ϕ and φ are elastic displacement vector, electric potential and magnetic potential; β_{ij}, γ_i and υ_i are thermal-stress constants, pyroelectric coefficients and pyromagnetic coefficients; k_{ij} is the thermal conductivity; c_{ijkm}, e_{ijk}, h_{ijk}, α_{ij}, κ_{ij} and μ_{ij} are the elastic moduli, piezoelectric coefficients, piezomagnetic coefficients, magnetoelectric coefficients, dielectric constants and magnetic permeability, respectively. The general solution to the Eq. (1) can be written as (Qin, 2005)

$$\begin{aligned} &T = g'(z_t) + \overline{g'(z_t)} \\ &\mathbf{u} = \mathbf{A}\mathbf{f}(z)\mathbf{q} + \mathbf{c}g(z_t) + \overline{\mathbf{A}\mathbf{f}(z)\mathbf{q}} + \overline{\mathbf{c}g(z_t)} \end{aligned} \tag{7}$$

with $\mathbf{A} = [A_1, \ A_2, \ A_3, \ A_4, A_5]$, $\mathbf{f}(z) = diag[f(z_1), \ f(z_2), \ f(z_3), \ f(z_4), \ f(z_5)]$, $\mathbf{q} = [q_1, \ q_2, \ q_3, \ q_4, \ q_5]^T$, $z_t = x_1 + p_* x_2$, $z_i = x_1 + p_i x_2$, in which the prime denotes differentiation with the argument, the overbars denote complex conjugation, $\mathbf{q}$ is a constant vector to be determined by the boundary conditions, g and $\mathbf{f}$ are arbitrary analytic function, p_*, p_i, $\mathbf{A}$ and $\mathbf{c}$ are constants determined by

$$\begin{aligned} &k_{11} + 2k_{12}p_* + k_{22}p_*^2 = 0 \\ &\left[\mathbf{Q} + p_i\left(\mathbf{R} + \mathbf{R}^{\mathrm{T}}\right) + p_i^2\mathbf{T}\right]\mathrm{A}_i = 0 \\ &\left[\mathbf{Q} + p_*\left(\mathbf{R} + \mathbf{R}^{\mathrm{T}}\right) + p_*^2\mathbf{T}\right]\mathbf{c} = \boldsymbol{\chi}_1 + p_*\boldsymbol{\chi}_2 \end{aligned} \tag{8}$$

in which superscript "T" denotes the transpose, $\boldsymbol{\chi}_i$, $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{T}$ are defined by

$$\begin{aligned} &\boldsymbol{\chi}_i = [\beta_{i1}, \ \beta_{i2}, \ \beta_{i3}, \ \gamma_i, \ \upsilon_i]^T, \quad \mathbf{Q}_{IK} = E_{1IK1}, \quad \mathbf{R}_{IK} = E_{1IK2}, \\ &\mathbf{T}_{IK} = E_{2IK2} \end{aligned} \tag{9}$$

The thermal flux, $\mathbf{h}$, and stress, electric displacement and magnetic induction (SEDMI), $\mathbf{\Pi}_{iJ}$, can be obtained from Eq. (2) as

$$\begin{aligned} &h_i = -(k_{i1} + p_*k_{i2})g''(z_t) - (k_{i1} + \overline{p_*}k_{i2})\overline{g''(z_t)} \\ &\mathbf{\Pi}_{1J} = -\mathbf{\Phi}_{J,2}, \quad \mathbf{\Pi}_{2J} = \mathbf{\Phi}_{J,1} \end{aligned} \tag{10}$$

where $\mathbf{\Phi}$ is the SEDMI function give as

$$\mathbf{\Phi} = \mathbf{B}\mathbf{f}(z)\mathbf{q} + \mathbf{d}g(z_t) + \overline{\mathbf{B}\mathbf{f}(z)\mathbf{q}} + \overline{\mathbf{d}}\overline{g(z_t)} \tag{11}$$

with

$$\begin{aligned} &\mathbf{B} = \mathbf{R}^T\mathbf{A} + \mathbf{TAP} \\ &\mathbf{P} = diag[p_1, \ p_2, \ p_3, \ p_4, \ p_5] \\ &\mathbf{d} = \left(\mathbf{R}^T + p_*\mathbf{T}\right)\mathbf{c} - \boldsymbol{\chi}_2 \end{aligned} \tag{12}$$