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Elastic field due to an edge dislocation in a multilayered composite

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ABSTRACT

A numerical procedure is presented for the analysis of the elastic field due to an edge dislocation in a multilayered composite. The multilayered composite consists of *n* perfectly bonded layers having different material properties and thickness, and two half-planes adhere to the top and bottom layers. The stiffness matrices for each layer and the half-planes are first derived in the Fourier transform domain, then a set of global stiffness equations is assembled to solve for the transformed components of the elastic field. Since the singular part of the elastic field corresponding to the dislocation in the full-plane has been extracted from the transformed components, regular numerical integration is needed only to evaluate the inverse Fourier transform. Numerical results for the elastic field due to an edge dislocation in a bimaterial medium are shown in fairly good agreement with analytical solutions. The elastic field and the Peach–Kohler image force are also presented for an edge dislocation in a single layered half-plane, a two-layered half-plane and a multilayered composite made of alternating layers of two different materials.

1. Introduction

The determination of elastic field due to an edge dislocation in a multilayer is important to study the effects of dislocations on the mechanical properties of multilayered materials and thin films. The misfit dislocations in epitaxial thin films at interfaces could provide stress relaxation mechanisms as the epitaxial films reach a critical thickness (Hirth and Feng, 1990; Freund, 1993). In metallic multilayered composites, the interaction between dislocations and interfaces can significantly increase the yield strength and hardness (Venkatraman and Bravman, 1992; Misra et al., 1998). The strengthening mechanisms are primary due to the dislocation pile-up at the interfaces for the metallic multilayer having the layer thickness at large length scales while the Orowan bowing and dislocation interactions for the layer thickness reduced to the nanometer range (Nix, 1998; Misra and Kung, 2001; Akashen et al., 2007; Wang and Misra, 2011). To study the effects of dislocations on the strengthening mechanisms of multilayered composites, it is essential to quantify the interactions between dislocations and interfaces.

Head (1953, 1960) first studied the screw dislocations interacting with a surface layer perfectly bonded to a half-plane. For the layer stiffer than the substrate, there exists a long-range attraction and a short-range repulsion on a dislocation in the substrate. The equilibrium position of the screw dislocations and the applied stress necessary to overcome the repulsion are also calculated numerically for different ratios of shear moduli. Weeks et al. (1968) considered an edge dislocation with a Burgers vector normal to the layer-substrate interface and presented an exact analysis for the elastic field and the Peach–Kohler force due to an edge dislocation in the substrate near the surface layer. The elastic field was obtained by the superposition of the solution of an edge dislocation in the joined half-plane and that of the reversed traction prescribed on the free surface of the layered half-plane. Lee and Dundurs (1973) extended analysis to the problems of an edge dislocation in the surface layer that adheres to a substrate. Kelly et al. (1995) used the same technique to calculate the stress field induced by an edge dislocation of arbitrary orientation in both the surface layer and substrate. For and edge dislocation in an infinite trimaterial medium, Choi and Earmme (2002a,b) used the complex potentials and the alternating technique to derive series solutions for the stress field induced by the dislocation for both isotropic and anisotropic materials. In recent years, the discrete dislocation dynamics have been applied to investigate the effects of individual dislocation on the deformation behavior of materials at small length scale (Van der Giessen and Needleman, 1995; Akashen et al., 2007). The elastic field due to the collected dislocations is obtained by the superposition of the field associated with individual dislocations in an infinite medium and the image field that corrects for the actual boundary conditions. The image field is commonly solved by the finite element method to satisfy the conditions on the boundaries, and the problem is generally limited to a single film on a rigid substrate.

While analytical solutions are mostly for the dislocation in either a bimaterial or a film-substrate medium, there are only

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few results presented for the dislocation in a multilayered medium with the layers having dissimilar properties and thickness. Ovecoglu et al. (1987) extended the Head's solution (1960) to the image analysis of a screw dislocation in thin films on substrate for up to the case of four interfaces. Kamat et al. (1987) conducted similar study on the screw dislocations in a multilayer structure made of alternating layers of two phases up to five layers. Wang et al. (2007) presented the image decomposition method to calculate the elastic field induced by a mixed dislocation in anisotropic multilayer medium. The problem is decomposed into homogeneous sub-problems with the distributed image dislocations introduced at the interfaces to satisfy the interface and free surface conditions which lead to a set of non-singular Fredholm integral equations of the second kind.

In this paper, a numerical approach based on the Fourier transform is presented to derive the displacement and stress fields induced by an edge dislocation in a multilayered composite which is made of perfectly bonded layers having different material properties and thickness. The stiffness matrices for each layer and the semi-infinite planes are first derived in the Fourier transform domain, then a global stiffness matrix for the multilayered composite is assembled to solve for the transformed components of the elastic field. As the singular part of the elastic field is extracted from the transformed components, regular numerical integration is needed only to evaluate the inverse Fourier transform. The elastic field and the Peach–Kohler image force are presented for an edge dislocation in a single layered half-plane, a two-layered half-plane and a multilayered composite made of alternating layers of two different materials.

2. Mathematical formulation

Consider the two-dimensional problem of an edge dislocation in a multilayered composite as shown in Fig. 1 where the layered medium consists of n dissimilar layers which are perfectly bonded to each other and adheres to half-planes at the upper and lower

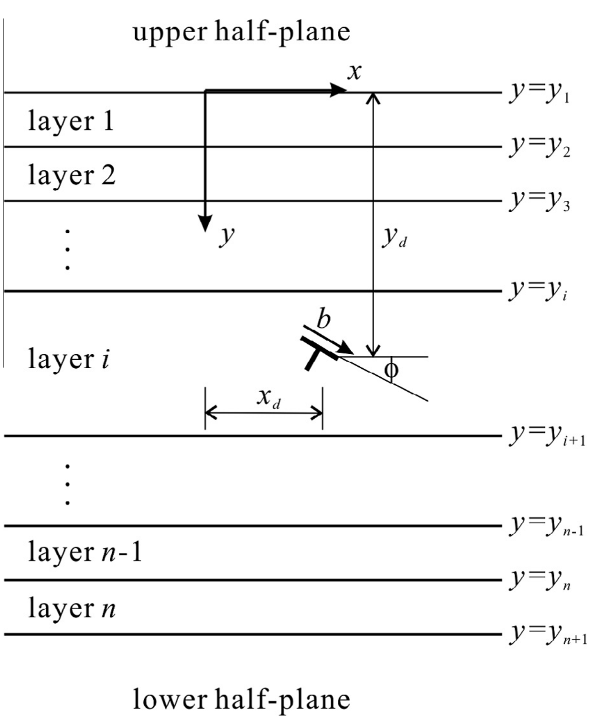

Fig. 1. An edge dislocation with a Burgers vector $\vec{b}$ on the slip plane making an angle of ϕ with the x-axis in a multilayered composite.

surfaces. The shear modulus and Poisson's ratio for the layers are denoted by (μ_i, υ_i), $i = 1, 2, \ldots, n$, and those for the upper and lower half-planes are denoted by (μ_u, υ_u) and (μ_l, υ_l), respectively. The edge dislocation with a burgers vector $\vec{b} = b_x\vec{i} + b_y\vec{j}$ and the dislocation line in the x_3-direction is located at the point (x_d, y_d) in the jth layer and its slip plane makes an angle of $\phi = \tan^{-1}(b_y/b_x)$ with the x-axis. The equilibrium and continuity conditions for the perfectly bonded interfaces are described as:

$$u_j^i(x, y_i) = u_j^{i+1}(x, y_i) \quad i = 1, 2, \ldots, n+1 \tag{1}$$

$$\sigma_{2j}^i(x, y_i) = \sigma_{2j}^{i+1}(x, y_i) \quad i = 1, 2, \ldots, n+1 \tag{2}$$

Here the subscript 'j', $j = 1, 2$, stands for x and y, respectively.

For isotropic materials, the displacement components for each layer, e.g., the ith layer, $y_i \leqslant y \leqslant y_{i+1}$, can be expressed in terms of Papkovich–Neuber potentials as:

$$u_x^i = -y\frac{\partial \varphi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \tag{3}$$

$$u_y^i = \kappa_i \varphi^i - y\frac{\partial \varphi^i}{\partial y} - \frac{\partial \psi^i}{\partial y} \tag{4}$$

where $\kappa_i = 3 - 4\upsilon_i$ for the plane strain case and the potential functions φ^i and ψ^i satisfy the Laplace equation: $\nabla^2\varphi^i = \nabla^2\psi^i = 0$. The general solution of the potential functions can be expressed by the Fourier integrals as (see Appendix A):

$$\varphi^i(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A_iE_i(y) + B_iE_{i+1}(y)]\frac{e^{ix\eta}}{\eta}d\eta \tag{5}$$

$$\psi^i(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[C_iE_i(y) + D_iE_{i+1}(y)]\frac{e^{ix\eta}}{\eta^2}d\eta \tag{6}$$

where $E_i(y) = e^{-|\eta(y-y_i)|}$, $E_{i+1}(y) = e^{-|\eta(y-y_{i+1})|}$ and A_i, B_i, C_i, and D_i are functions of η. By substituting Eqs. (5) and (6) into Eqs. (3) and (4) and using the Hooke's law, the displacement and stress components can be written as:

$$u_x^i = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\left(-yA_i - \frac{C_i}{\eta}\right)E_i(y) + \left(-yB_i - \frac{D_i}{\eta}\right)E_{i+1}(y)\right]ie^{ix\eta}d\eta \tag{7}$$

$$\begin{aligned} u_y^i = \frac{1}{2\pi}\int_{-\infty}^{\infty}&\left\{\left[(\kappa_i + y|\eta|)A_i + \frac{|\eta|}{\eta}C_i\right]E_i(y)\right.\\ &\left.+\left[(\kappa_i - y|\eta|)B_i - \frac{|\eta|}{\eta}D_i\right]E_{i+1}(y)\right\}\frac{e^{ix\eta}}{\eta}d\eta \end{aligned} \tag{8}$$

$$\begin{aligned} \sigma_{xx}^i = \frac{\mu_i}{\pi}\int_{-\infty}^{\infty}&\left\{\left[-\frac{(3-\kappa_i)|\eta|}{2\eta}A_i + y\eta A_i + C_i\right]E_i(y)\right.\\ &\left.+\left[\frac{(3-\kappa_i)|\eta|}{2\eta}B_i + y\eta B_i + D_i\right]E_{i+1}(y)\right\}e^{ix\eta}d\eta \end{aligned} \tag{9}$$

$$\begin{aligned} \sigma_{yy}^i = \frac{\mu_i}{\pi}\int_{-\infty}^{\infty}&\left\{\left[-\frac{(\kappa_i+1)|\eta|}{2\eta}A_i - y\eta A_i - C_i\right]E_i(y)\right.\\ &\left.+\left[\frac{(\kappa_i+1)|\eta|}{\eta}B_i - y\eta B_i - D_i\right]E_{i+1}(y)\right\}e^{ix\eta}d\eta \end{aligned} \tag{10}$$

$$\begin{aligned} \sigma_{xy}^i = \frac{\mu_i}{\pi}\int_{-\infty}^{\infty}&\left\{\left[\frac{(\kappa_i-1)}{2}A_i + y|\eta|A_i + \frac{|\eta|}{\eta}C_i\right]E_i(y)\right.\\ &\left.+\left[\frac{(\kappa_i-1)}{2}B_i - y|\eta|B_i - \frac{|\eta|}{\eta}D_i\right]E_{i+1}(y)\right\}ie^{ix\eta}d\eta \end{aligned} \tag{11}$$

In Eqs. (7)–(11), the elastic field in each layer requires the solution of the functions: A_i, B_i, C_i, and D_i, which are determined from the continuity and equilibrium conditions at the interfaces. To evaluate the unknown functions in the Fourier transformed domain, the