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Perturbation analysis for an imperfect interface crack problem using weight function techniques

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ABSTRACT

We analyse a problem of anti-plane shear in a bi-material plane containing a semi-infinite crack situated on a soft imperfect interface. The plane also contains a small thin inclusion (for instance an ellipse with high eccentricity) whose influence on the propagation of the main crack we investigate. An important element of our approach is the derivation of a new weight function (a special solution to a homogeneous boundary value problem) in the imperfect interface setting. The weight function is derived using Fourier transform and Wiener–Hopf techniques and allows us to obtain an expression for an important constant $\sigma_0^{(0)}$ (which may be used in a fracture criterion) that describes the leading order of tractions near the crack tip for the unperturbed problem. We present computations that demonstrate how $\sigma_0^{(0)}$ varies depending on the extent of interface imperfection and contrast in material stiffness. We then perform perturbation analysis to derive an expression for the change in the leading order of tractions near the tip of the main crack induced by the presence of the small defect, whose sign can be interpreted as the inclusion's presence having an amplifying or shielding effect on the propagation of the main crack.

1. Introduction

In this paper we present a method to evaluate important constants which describe the behaviour of physical fields near crack tips in a perturbed problem set in a domain containing an imperfect interface.

Imperfect interfaces account for the fact that the interface between two materials is almost never sharp. Atkinson (1977) accounted for this observation by placing a very thin strip of a homogeneous material in the model between two larger bodies with different elastic moduli to that of the strip. If the thin layer is considered to be either much softer or stiffer than the main bodies, its presence can be replaced in models by transmission conditions, whose derivation can be found for example in Antipov et al. (2001) for a soft imperfect interface, or Mishuris et al. (2006) for a stiff imperfect interface. We shall consider only soft imperfect interfaces in the present paper.

Klarbring and Movchan (1998) presented an asymptotic model of adhesive joints in a layered structure. Mishuris (2001) found the asymptotic behaviour of displacements and stresses in a vicinity of the crack tip situated on a soft imperfect interface between two different elastic materials, where the non-ideal interface is replaced by non-ideal transmission conditions. For such a case, the asymptotics are of a markedly different form to the perfect interface case, in which components of stress exhibit a square root singularity at the crack tip; such behaviour is not present for imperfect interface cracks.

A key element of our approach will be the derivation of a new weight function. The concept of weight functions was introduced by Bueckner (1970). In the perfect interface setting these provide weights for the loads applied to the crack surfaces such that their weighted integrals over the crack surfaces provide the stress intensity factors at a certain point. Vellender et al. (2011) modified the weight function technique to yield similarly useful asymptotic constants that characterise stress fields near crack tips along an imperfect interface.

A survey of macro–microcrack interaction problems can be found in Petrova et al. (2000). Of particular relevance is the recent manuscript of Mishuris et al. (2011) which examines an analogous problem to that presently considered with a perfect interface in place of the imperfect interface. The approach in that paper utilises the dipole matrix approach of Movchan and Movchan (1995) to construct an asymptotic solution that takes into account the presence of a micro-defect such as a small inclusion. The present paper seeks to adapt this approach to the imperfect interface setting.

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2. Structure and summary of main results

We adopt the following structure for the paper. We first formulate the physical problem before giving the weight function problem formulation. Fourier transform techniques allow us to obtain a Wiener–Hopf type problem for the weight function, whose kernel we factorise in a computationally convenient fashion. The Wiener–Hopf equation is solved to yield expressions for the weight function and comparisons are drawn between the perfect and imperfect interface weight function problems.

We then use the reciprocal theorem (Betti formula) in the spirit of Willis and Movchan (1995) to relate the sought physical solution to the weight function. The presence of imperfect interface transmission conditions alters properties of the functions in the Betti identity and so different analysis is required. The application of Betti's identity enables us to find an expression for the leading order of tractions $\sigma_0^{(0)}$ near the crack tip in terms of the new weight function and the imposed arbitrary tractions prescribed on the faces of the crack:

$$\sigma_0^{(0)} = \frac{1}{2}\sqrt{\frac{\mu_0}{\pi}}\int_{-\infty}^{\infty}\xi([\![\bar{U}]\!](\xi)\langle\bar{p}\rangle(\xi) + \langle\bar{U}\rangle(\xi)[\![\bar{p}]\!](\xi))\,\mathrm{d}\xi. \quad (1)$$

Here, bars denote Fourier transform, μ_0 is a constant depending on the material parameters and extent of interface imperfection, $[\![U]\!]$ and $\langle U\rangle$ are respectively the jump and average of the weight function across the crack/interface line, and $[\![p]\!]$ and $\langle p\rangle$ are the jump and average of the tractions prescribed on the crack faces.

In Section 7, we perform perturbation analysis to determine the impact on the tractions near the crack tip of the presence of a small inclusion. The asymptotic solution is sought in the form

$$u(\boldsymbol{x},\varepsilon) = u^{(0)}(\boldsymbol{x}) + \varepsilon W^{(1)}(\xi) + \varepsilon^2 u^{(1)}(\boldsymbol{x}) + o(\varepsilon^2), \quad \varepsilon \to 0, \quad (2)$$

where $u^{(0)}$ is the unperturbed physical displacement solution (the solution with no inclusion present), $\varepsilon W^{(1)}$ is a boundary layer concentrated near the inclusion and $\varepsilon^2 u^{(1)}$ is introduced to fulfil the original boundary conditions on the crack faces and along the imperfect interface. This enables us to find the first order variation in the crack tip tractions; we expand the constant σ_0 as

$$\sigma_0 = \sigma_0^{(0)} + \varepsilon^2\Delta\sigma_0 + o(\varepsilon^2), \quad \varepsilon \to 0, \quad (3)$$

and use Betti identity arguments to derive an expression for $\Delta\sigma_0$ (see (138)). This is interpreted physically as the change in traction near the crack tip induced by the inclusion's presence; as such we say that the sign of $\Delta\sigma_0$ for any given positioning and configuration of the inclusion either shields or amplifies the propagation of the main crack. Note that for the unpeturbed setup (with no inclusion present) $\sigma_0 = \sigma_0^{(0)}$ and so we will naturally drop the superscript when referring to the quantity corresponding to the unperturbed problem.

We conclude the paper by presenting numerical results in Section 9. In particular we show how $\sigma_0^{(0)}$ varies depending on the extent of interface imperfection and choice of material contrast parameter for different loadings. These computations are performed for point loadings that are chosen to be illustrative of the suitability of our method to asymmetric self-balanced loadings. We further propose a method of comparing $\sigma_0^{(0)}$ with stress intensity factors from the analogous perfect interface problem and find agreement as the extent of interface imperfection tends towards zero. We also present computations that show the sign of $\Delta\sigma_0$ for varying location and orientation of the micro-defect.

3. Formulation of physical and weight function problems

3.1. Physical formulation

We consider an infinite two-phase plane with an imperfect interface positioned along the positive x-axis. A semi-infinite crack is placed occupying the line $\{(x,y): x<0, y=0\}$. We refer to the half-planes above and below the crack and interface respectively as $\Pi^{(1)}$ and $\Pi^{(2)}$. The material occupying $\Pi^{(j)}$ has shear modulus μ_j and mass density ρ_j for $j=1,2$. The anti-plane shear displacement function u satisfies the Laplace equation

$$\nabla^2 u(x,y) = 0. \quad (4)$$

The plane also contains a micro-defect whose centre is at the point $\boldsymbol{Y}$; we will consider in particular elliptic inclusions although other types of defect may be incorporated into the model provided a suitable dipole matrix can be obtained (see for example Mishuris et al., 2011 in which micro-cracks and rigid line inclusions are considered). The defect g_ε has shear modulus μ_{in}, is placed at a distance d from the crack tip, makes an angle ϕ with the imperfect interface and is oriented at an angle α to the horizontal as shown in Fig. 1. The value of μ_{in} may be greater than or less than the value of μ_{out} (which may be μ_1 or μ_2 depending where the defect is placed), and so both stiff and soft defects can be considered.

We assume continuity of tractions across the crack and interface, and introduce imperfect interface conditions ahead of the crack:

$$\mu_1\frac{\partial u}{\partial y}\bigg|_{y=0+} = \mu_2\frac{\partial u}{\partial y}\bigg|_{y=0-}, \quad x>0, \quad (5)$$

$$[\![u]\!] - \kappa\mu_1\frac{\partial u}{\partial y}\bigg|_{y=0+} = 0, \quad x>0, \quad (6)$$

where the notation $[\![u]\!]$ defines the jump in displacement across $x=0$, i.e.

$$[\![u]\!](x) = u_1(x,0^+) - u_2(x,0^-). \quad (7)$$

The parameter $\kappa>0$ describes the extent of imperfection of the interface, with larger κ corresponding to more imperfect interfaces. We further impose prescribed tractions $p_\pm$ on the crack faces:

$$\mu_1\frac{\partial u}{\partial y}\bigg|_{y=0+} = p_+(x), \quad \mu_2\frac{\partial u}{\partial y}\bigg|_{y=0-} = p_-(x); \quad x<0. \quad (8)$$

These tractions are assumed to be self-balanced; that is

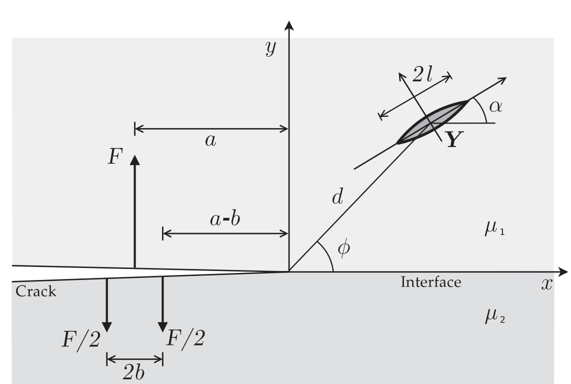

Fig. 1. Geometry for the physical setup. The crack tip is placed at the origin of an infinite plane composed of materials with shear modulus μ_j occupying half-planes $\Pi^{(j)}$ above and below the crack and imperfect interface for $j=1,2$. The central point **Y** of a micro-defect is situated at a distance d from the tip of the main crack.