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Carbon fiber-reinforced rectangular isolators with compressible elastomer: Analytical solution for compression and bending

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ABSTRACT

The behavior of multilayer elastomeric isolators employing carbon fibers as reinforcement material rather than steel has been considered. This kind of reinforcement is used to make the isolators lighter and cheaper, since carbon fibers (or Kevlar) are much more resistant per unit weight than steel. From the mechanical point of view, the main difference is that the fiber reinforcement cannot be considered rigid in extension as it is usually done for steel plates. In this paper an analytical model to analyze the compression and bending behavior of fiber-reinforced rectangular isolators is presented. The model takes into account, for the first time, both the reinforcement extensibility and the compressibility of the elastomer. An analytical solution to predict deformations, stresses and stiffness is here determined in terms of Fourier series, both for compression and bending.

1. Introduction

Seismic isolation is a design approach to preserve structures from earthquakes loads, which is based on the considerably increase of the fundamental vibration period of structures due to isolators presence. This goal is essentially reached by utilizing devices which exhibit significant horizontal flexibility, maintaining high vertical stiffness. Steel reinforced elastomeric isolators (SREIs), usually used for seismic isolation, consist of a number of sheets of elastomeric layers alternated to thin steel plates and bonded to them. Steel plates are supposed to be rigid and consequently lateral displacements at the top and the bottom of each elastomeric layer are zero, thus increasing computed vertical stiffness. Two steel plates, much thicker than others, are positioned at the top and bottom of the whole isolator; they allow the isolator to be rigidly connected, by means of other two thick steel plates, to the foundation and to the superstructure. This kind of isolators is heavy and expensive and consequently this isolation technology is generally applied in specific cases only (emergency centers, hospitals or large-expensive buildings).

In the last years, theoretical studies have been carried out to conceive isolators lighter and possibly made by a less labor-intensive manufacturing process. Kelly (2002) and Kelly and Takhirov (2002) showed that such kind of isolators can be produced, replacing steel plates with carbon fiber sheets or Kevlar. In the following years, experimental and numerical studies (Kelly, 2002; Kelly and Takhirov, 2002; Moon et al., 2003; Russo et al., 2008; Toopchi-Nezhad et al., 2008; Mordini and Strauss, 2008; Gerhaher et al., 2009) have been carried out to verify the feasibility and reliability of such isolators.

The great advantage of this kind of isolators is that they are much lighter weighting, but fiber reinforcement exhibits extensional flexibility producing a small but significant reduction in isolators vertical stiffness, with respect to steel reinforced isolators. Therefore a good design of isolators requires a correct estimation of vertical and flexural stiffness.

Some models to evaluate vertical and flexural stiffness for carbon fiber reinforced elastomeric isolators are already present in the literature. They all assume both the elastomeric material and the fiber-reinforcement to be linearly elastic; this is allowed by the fact that displacements involved by compression and bending actions are small. Moreover, the stress state is supposed to be dominated by internal pressure (Kelly, 1997). In order to obtain analytical solutions, identical deformations are assumed for each elastomeric reinforced layer. By this way, the analysis of the overall isolator reduces on the analysis of the single fiber-reinforced elastomeric layer.

Different models can be considered, according to the different hypothesis that can be introduced on the geometry and on the mechanical properties of the materials which constitute the isolator. In particular, (i) the isolator shape can be circular, rectangular or an infinitely long strip; (ii) the reinforcement can be assumed to be rigid (typically for steel reinforcement) or extensible (fiber-reinforcement) and (iii) the elastomer can be assumed incompressible or not.

The first proposed model (Kelly, 1997) considers isolators with rigid reinforcement and compressible or incompressible rubber.

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Kelly (1999) takes into account reinforcement extensibility for infinitely long isolators. For the same shape, Kelly and Takhirov (2002) considers also rubber compressibility. Tsai and Kelly (2002) study the behavior of rectangular isolators, taking into account reinforcing extensibility but neglecting rubber compressibility; they consider the reinforcement as a homogenous isotropic elastic sheet supporting shear stresses and obeying Poisson effect.

On the basis of the above mentioned approaches, in this paper an analytical model for rectangular carbon fiber-reinforced isolators, which takes into account reinforcement flexibility and rubber compressibility, is for the first time presented. In this case, the reinforcement is made by fabrics constituted with carbon fibers laid out in two orthogonal directions and simply overlapped. As it is explained in detail in the following chapter, such a kind of material does not present Poisson effect and it is not able to sustain shear stress. For this reason the reinforcement model is different from Tsai and Kelly's (2002) one.

The analytical solution for compression and bending is given in terms of Fourier series. To validate the model a Finite Element (FE) analysis has been also carried out; it showed an excellent agreement with the analytical solution.

As all above mentioned models do, the proposed one assumes a linear elastic behavior both for elastomer and the reinforcement. For this reason, the range of applicability of the obtained solution is restricted to small deformations occurring under service loading. Being the assumed behavior elastic, no damping effect can be taken into account.

Linear behavior for the materials is also assumed in the FE analysis to make the numerical model consistent with the analytical one.

2. Compression analysis

A rectangular isolator with extensible reinforcement is a system of elastomeric layers alternated with reinforcing fiber fabrics. Thickness of each elastomeric layer is t, while the ideal equivalent thickness of the fiber fabric is t_f, being the fibers volume assumed to be uniformly distributed on the whole isolator cross section. In Fig. 1 a portion of an isolator bounded by the symmetry vertical plane and constituted by three elastomeric layers, is represented.

To study the static behavior of the isolator, a single fiber-reinforced elastomeric layer extracted from the whole, as shown in Fig. 1, is considered. The single fiber-reinforced elastomeric layer is made by an elastomeric layer and by two superior and inferior fiber sheets of ideal thickness $t_f/2$ each, as represented in Fig. 2. The side length parallel to the x axis is $2b$ and to the y axis is $2h$.

2.1. Equilibrium equations in the elastomeric layer

A vertical compressive load P is supposed to be applied to the isolator layer. Each point of the elastomer exhibits the displacements u, v, and w in x, y and z coordinate directions respectively.

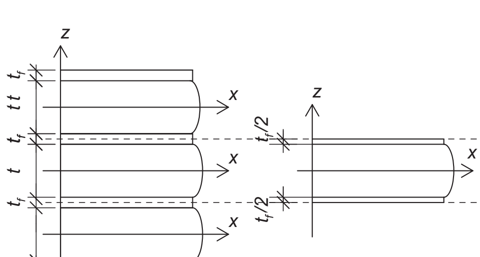

Fig. 1. Portion of the isolator and the extracted single layer.

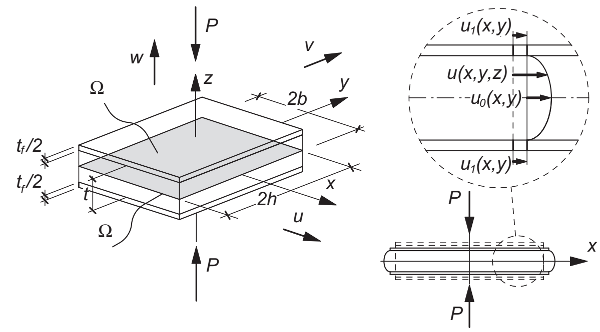

Fig. 2. Single fiber-reinforced elastomeric layer under compression and the displacements field.

By assuming fiber sheets to be perfectly bonded to the elastomeric layers, these displacements can be expressed as (Tsai and Kelly, 2002):

$$u(x,y,z) = u_0(x,y)(1 - 4z^2/t^2) + u_1(x,y) \quad (1)$$

$$v(x,y,z) = v_0(x,y)(1 - 4z^2/t^2) + v_1(x,y) \quad (2)$$

$$w(x,y,z) = w(z) \quad (3)$$

First terms on the right-hand side of Eqs. (1) and (2), represent the kinematic assumption that the displacements u and v vary quadratically along z, u_0 and v_0 being the displacements of the elastomeric layer mean plane x–y. The additional displacements u_1 and v_1, which are constant through the thickness, are the displacements evaluated at the fiber reinforcement interface. These displacements are equals to those of the fiber sheet, due to perfect bonding between elastomer and fibers. The fiber sheet displacements u_1 and v_1 are assumed to be constant along the fiber sheet thickness, due to its smallness. Eq. (3) represents the kinematic assumption that horizontal planes remain plane after the deformation. A sketch of the displacement field is shown on the right-hand side in Fig. 2.

Accordingly to the hypothesis introduced by Kelly (1997), the stress state in the elastomer is supposed to be dominated by the internal pressure p, such that the corresponding stress components are assumed to be:

$$\sigma_{xx} \approx \sigma_{yy} \approx \sigma_{zz} \approx -p \quad (4)$$

$$\tau_{xy} = \tau_{yx} \approx 0 \quad (5)$$

The equilibrium equations in x and y directions for the stresses in the elastomer are therefore reduced to (Kelly, 1997)

$$-p_{,x} + \tau_{xz,z} = 0 \quad (6)$$

$$-p_{,y} + \tau_{yz,z} = 0 \quad (7)$$

where commas imply partial differentiation with respect to the coordinate indicated after the comma.

A linear elastic behavior for the elastomer is assumed.

By taking into account the compressibility of the material, the compressibility equation can be written

$$u_{,x} + v_{,y} + w_{,z} = -\frac{p}{K} \quad (8)$$

where K is the bulk modulus of the elastomer. The right-hand side in Eq. (8) is equal to zero when the elastomer compressibility is not taken into account, as was done by Tsai and Kelly (2002).

Eqs. (1)–(8) are the governing equations of the elastic problem of rectangular isolators with extensible reinforcement and compressible elastomer. A solution of this problem is here obtained for the first time as shown in the following paragraphs.