International Journal of Solids and Structures 50 (2013) 3634–3648

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Strain localization and fabric evolution in sand

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ARTICLE INFO

Article history:
Received 20 February 2013
Received in revised form 2 May 2013
Available online 18 July 2013

Keywords:
Shear band
Anisotropy
Fabric evolution
Strain localization
Plane strain
Sand

ABSTRACT

Strain localization is frequently observed in sand and is considered an important precursor related to major geohazards such as landslides, debris flow and failure of relevant geo-structures. This paper presents a numerical study on strain localization in sand, with a special emphasis on the influence of soil fabric and its evolution on the initiation and development of shear band. In particular, a critical state sand plasticity model accounting for the effect of fabric and its evolution is used in the finite element analysis of plane strain compression tests. It is found that the initiation of shear band is controlled by the initial fabric, while the development of shear band is governed by two competing physical mechanisms, namely, the structural constraint and the evolution of fabric. The evolution of fabric generally makes the sand response more coaxial with the applied load, while the structural constraint induced by the sample ends leads to more inhomogeneous deformation within the sand sample when the initial fabric is non-coaxial with the applied stress. In the case of smooth boundary condition, structural constraint dominates over the fabric evolution and leads to the formation of a single shear band. When the boundary condition is rough, the structural constraint may play a comparable role with fabric evolution, which leads to symmetric cross-shape shear bands. If the fabric is prohibited from evolving in the latter case, a cross-shape shear band pattern is found with the one initiated first by the structural constraint dominating over the second one. In all cases, significantly larger dilation and fabric evolution are observed inside the shear band than outside. The simulated shear band orientation coincides with the Roscoe's angle for cases with high confining pressure and lies in between the Roscoe's angle and Arthur's angle for the low confining pressure cases.

1. Introduction

Strain localization is frequently observed in sand and is considered an important precursor of the failure of soil and relevant geostructures including major geohazards such as landslides and debris flow. Due to its apparent importance, relevant studies on strain localization, both experimentally and theoretically, have been an active area in geomechanics for decades (see, e.g., Chu et al., 1996; Vardoulakis, 1996; Mokni and Desrues, 1998; Desrues and Viggiani, 2004; Rechenmacher, 2006; Daouadji et al., 2011). These studies identified the following key factors which influence the strain localization in sand, including the density of sand, confining pressure, boundary conditions, sample size, imperfection and the drainage conditions. However, relatively less attention has been paid to the correlation of strain localization with the presence of fabric and its evolution in sand. The fabric in sand, or the so-call internal structure of sand attributable to the sand particle orientation, contact normal distribution and void space distribution, has been widely regarded to affect the key behavior of sand including dilatancy, liquefaction and critical state, and may influence the behavior of strain localization in sand in an important manner as well. Indeed, based on plain strain compression tests, Tatsuoka et al. (1990) found that the shear band development was indeed dependent on the initial bedding plane orientation, or the fabric, of the sample. In their torsional shear tests, the horizontal bedding plane in the sand deposit was also shown to act as attractor to shear band. Similar observations were further confirmed by Lade et al. (2008). A more recent numerical study by Fu and Dafalias (2011) based on discrete element method with elliptical granular particles also indicated that the development of shear band depends crucially on the initial fabric orientation.

Meanwhile, it is important to realize that the fabric in sand is not stationary and may evolve constantly over different deformation stages of sand. The change of internal structure in soil due to fabric evolution may apparently affect the initiation and development of shear band. This has indeed been proved by micromechanics-based studies including distinct element simulations (e.g., Bardet and Proubet, 1991; Oda and Iwashita, 2000; Evans and Frost, 2010; Chupin et al., 2011; Fu and Dafalias, 2011; Zhao and Guo, 2013a,b; Guo and Zhao, 2013; Zhao et al., 2013) and physical tests on photoelastic rods (Oda et al., 1982; Oda and Kazama, 1998). For example, based on plane strain compression tests on sand and biaxial compression tests on rod-like particles, Oda and Iwashita (2000) have

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http://dx.doi.org/10.1016/j.ijsolstr.2013.07.005

shown that during the strain hardening process of the tested samples, a column-like fabric structure was found growing in parallel to the major principal stress direction; this structure can then be gradually changed and be totally reconstructed in the softening process, especially inside the shear band. The 2D DEM simulations of direct shear and biaxial compression tests by Fu and Dafalias (2011) using elliptical particles demonstrate that critical states can be reached at very large local shear strain within a shear band and the critical state fabric is strongly anisotropic. Indeed, as pointed out by Li and Dafalias (2012), Zhao and Guo (2013a,b) and Guo and Zhao (2013), as an essential component in addition to void ratio and stress, fabric anisotropy has to be carefully taken into account in the characterization of critical state in sand, and the soil fabric is highly anisotropic at critical state. Meanwhile, since a sample can be initially isotropic, its fabric has to experience substantial change before reaching an anisotropic critical state fabric, which highlights the importance of considering fabric evolution.

Towards the investigation of strain localization in sand, it is hence crucial to properly consider the role of fabric and fabric evolution. This is unfortunately absent in most existing studies including those derived from classic bifurcation theory (e.g., Vardoulakis, 1980,1996; Lade, 2003; Hashiguchi and Tsutsumi, 2007; Gutierrez, 2011) and those based on finite element method (e.g., Shuttle and Smith, 1988; Anand and Gu, 2000; Tejchman and Górski, 2010). Inherent fabric anisotropy has recently been considered in a hypoplastic model by Bauer et al. (2004) and Tejchman et al. (2007) in the simulation of shear band development in sand. While their studies realistically capture the dependence of shear band thickness and inclination on the initial bedding plane orientation of the sand sample, the interplay between fabric evolution and the development of shear band as two distinctive physical processes has not been properly considered. This issue will be carefully addressed in this paper. In this study, an anisotropic sand model newly developed by the authors (Gao et al., 2013) will be employed in conjunction with finite element method to investigate the strain localization in sand under plane strain compression. With a fabric evolution law embedded in this model, the effect of fabric and its evolution on the development of shear band in sand will be particularly highlighted. Based on detailed comparison between the numerical simulations and laboratory observations, the study offers further insights into the micromechanical basis of dilatancy and fabric evolution in shear bands. An interesting explanation of competing mechanisms between fabric evolution and structural constraint in the shear band development of sand under plane strain compression is also provided.

2. A critical state sand model accounting for fabric evolution

2.1. Model formulation

The model is developed based on the anisotropic critical state theory proposed recently by Li and Dafalias (2012). Detailed formulation, calibration and verification of the model at element test level can be found in Gao et al. (2013). The model features the following salient ingredients: (a) the employment of a void-based void fabric tensor which is more appropriate for describing the micromechanical basis of sand dilatancy than other fabric tensors (Li and Li, 2009); (b) accounting for fabric evolution and its effect on sand response; (c) capable of characterizing the non-coaxial sand behavior in a natural manner due to the consideration of fabric anisotropy and its evolution. Specifically, the sand model employs a yield function explicitly including the fabric anisotropy as follows

$$f = \frac{R}{g(\theta)} - He^{-k_h(A-1)^2} = 0 \tag{1}$$

where $R = \sqrt{3/2 r_{ij} r_{ij}}$ with $r_{ij} = (\sigma_{ij} - p\delta_{ij})/p = s_{ij}/p$ being the stress ratio tensor, in which σ_{ij} is the stress tensor, $p = \sigma_{ii}/3$ is the mean normal stress, δ_{ij} is the Kronecker delta and s_{ij} is the deviator stress; H is a hardening parameter; k_h is a non-negative model constant and $g(\theta)$ is an interpolation function based on the Lode angle θ of r_{ij} or s_{ij} as follows

$$g(\theta) = \frac{\sqrt{(1+c^2)^2 + 4c(1-c^2)\sin 3\theta} - (1+c^2)}{2(1-c)\sin 3\theta} \tag{2}$$

where $c = M_e/M_c$, the ratio between the critical state stress ratio R in triaxial extension M_e and that in triaxial compression M_c.

An important inclusion in the yield function in Eq. (1) is a fabric anisotropy variable A defined by the following joint invariant of the deviatoric void-based fabric tensor F_{ij} (Li and Li, 2009) and the loading direction tensor n_{ij} (Li and Dafalias, 2012; Dafalias et al., 2004)

$$A = F_{ij} n_{ij} \tag{3}$$

where F_{ij} is a symmetric traceless tensor whose norm $F = \sqrt{F_{ij}F_{ij}}$ is referred to as the degree of fabric anisotropy. For convenience, F_{ij} is normalized such that at the critical state, F is unity. For an initially cross-anisotropic sample with the isotropic plane being the x–z plane and the deposition direction aligning with the y-axis (e.g., Fig. 3(a) with the bedding plane orientation $\alpha = 0^\circ$), F_{ij} can be expressed as below

$$F_{ij} = \begin{pmatrix} F_{yy} & 0 & 0 \\ 0 & F_{xx} & 0 \\ 0 & 0 & F_{zz} \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} F_0 & 0 & 0 \\ 0 & -F_0/2 & 0 \\ 0 & 0 & -F_0/2 \end{pmatrix} \tag{4}$$

where F_0 ($\geqslant 0$) is the initial degree of anisotropy. If the initial bedding plane orientation does not align with the coordinate system (e.g., $\alpha \neq 0^\circ$ in Fig. 3(a)), the initial fabric tensor F_{ij} can be obtained by an orthogonal transformation of Eq. (4). Note that the fabric tensor will be manipulated directly in the sequel rather than dealing with the bedding plane angle and F_0 separately. The deviatoric unit loading direction tensor n_{ij} in Eq. (3) is defined as follows (Li and Dafalias, 2004)

$$n_{ij} = \frac{N_{ij} - N_{kk}\delta_{ij}/3}{|N_{ij} - N_{kk}\delta_{ij}/3|} \quad \text{with } N_{ij} = \frac{\partial \tilde{f}}{\partial r_{ij}} \tag{5}$$

where $\tilde{f} = R/g(\theta)$. Obviously, $n_{ii} = 0$ and $n_{ij}n_{ij} = 1$.

The evolution laws for H and F_{ij} are expressed as below

$$dH = \langle L \rangle r_h = \langle L \rangle \frac{G(1 - c_h e)}{pR} [M_c g(\theta) e^{-n\zeta} - R] \tag{6a}$$

$$G = G_0 \frac{(2.97 - e)^2}{1 + e} \sqrt{p p_a} \tag{6b}$$

$$dF_{ij} = \langle L \rangle \Theta_{ij} = \langle L \rangle k_f (n_{ij} - F_{ij}) \tag{7}$$

where $\langle\,\rangle$ denote the Macauley brackets with $\langle x \rangle = x$ for $x > 0$ and $\langle x \rangle = 0$ for $x \leqslant 0$; L is the loading index; e is the void ratio; c_h, n, G_0 and k_f are non-negative model parameters; G is the elastic shear modulus; p_a (=101 kPa) is the atmospheric pressure; ζ is the dilatancy state parameter defined as follows (Li and Dafalias, 2012)

$$\zeta = \psi - e_A(A - 1) \tag{8}$$

where e_A is a model parameter; $\psi = e - e_c$ is the state parameter defined by Been and Jefferies (1985) with e_c being the critical state void ratio corresponding to the current mean normal stress p. In the present work, the critical state line in the e–p plane is given by (Li and Wang, 1998)

$$e_c = e_\Gamma - \lambda_c (p/p_a)^\xi \tag{9}$$