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A domain-independent interaction integral for magneto-electro-elastic materials

Hongjun Yu [a,b,*], Linzhi Wu [a], Hui Li [b]

[a] Institute of Applied Mathematics, Harbin Institute of Technology, Harbin 150001, China
[b] School of Civil Engineering, Harbin Institute of Technology, Harbin 150090, China

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ABSTRACT

Magneto-electro-elastic (MEE) materials usually consist of piezoelectric (PE) and piezomagnetic (PM) phases. Between different constituent phases, there exist lots of interfaces with discontinuous MEE properties. Complex interface distribution brings a great difficulty to the fracture analysis of MEE materials since the present fracture mechanics methods can hardly solve the fracture parameters efficiently of a crack surrounded by complex interfaces. This paper develops a new domain formulation of the interaction integral for the computation of the fracture parameters including stress intensity factors (SIFs), electric displacement intensity factor (EDIF) and magnetic induction intensity factor (MIIF) for linear MEE materials. The formulation derived here does not involve any derivatives of material properties and moreover, it can be proved that an arbitrary interface in the integral domain does not affect the validity and the value of the interaction integral. Namely, the interaction integral is domain-independent for material interfaces and thus, its application does not require material parameters to be continuous. Due to this advantage, the interaction integral becomes an effective approach for extracting the fracture parameters of MEE materials with complex interfaces. Combined with the extended finite element method (XFEM), the interaction integral is employed to solve several representative problems to verify its accuracy and domain-independence. Good results show the effectiveness of the present method in the fracture analysis of MEE materials with continuous and discontinuous properties. Finally, the particulate MEE composites composed of PE and PM phases are considered and four schemes of different property-homogenization level are proposed for comparing their effectiveness.

1. Introduction

Magneto-electro-elastic (MEE) materials were first observed by Van Suchtelen (1972) and Van Run et al. (1974) who found that the ferrite-ferroelectric composites possessing both piezoelectric (PE) and piezomagnetic (PM) phases exhibited a magneto-electric coupling effect. Possessing the ability of converting mechanical, electric and magnetic energy, MEE materials have drawn significant interest in several engineering fields as a class of important functional materials, such as magnetic field probes, electronic packaging, hydrophones, medical ultrasonic imaging, actuators, waveguides, sensors, phase inverters, transducers (Wu and Huang, 2000; Ma et al., 2012). However, a great drawback of MEE materials is their inherent brittleness and low fracture toughness (Sladek et al., 2011). Generally, these materials may fail prematurely in service due to some defects such as cracks and holes, arising during the manufacturing process and subsequent handling. For this reason, it is of great important to understand the fracture feature of MEE materials.

On the theoretical side, Liu et al. (2001) studied Green's functions for MEE materials involving a crack. Based on the extended Stroh formalism, Wang and Mai (2003) obtained a general two-dimensional (2D) solution of the MEE field around the crack tip. Gao et al. (2003, 2004) obtained the explicit solutions in closed forms for a crack in MEE solids. Song and Sih (2003) examined the crack initiation and growth behavior in a MEE body. Subsequently, considerable research work was carried out on the static and dynamic fracture problems of MEE materials (Wang and Mai, 2004, 2007; Chen et al., 2004; Li, 2005; Hu and Li, 2005; Yong and Zhou, 2007; Guo and Lu, 2010; Zhang, 2011; Zhong, 2011; Ma et al., 2012). MEE materials usually contain PE and PM phases and the interfaces between constituent phases may reduce the reliability of MEE materials since the interfaces generally act as sources of failures in service. In order to improve the reliability, researchers proposed the concept of functionally graded materials (FGMs), a category of non-homogeneous materials with properties varying continuously, and recently, the concept of FGMs is extended to MEE materials, called functionally graded MEE (FGMEE)

* Corresponding author at: Institute of Applied Mathematics, Harbin Institute of Technology, Harbin 150001, China. Tel./fax: +86 451 86403725.
E-mail address: yuhongjun@hit.edu.cn (H. Yu).

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materials. The cracks in a FGMEE solid subjected to anti-plane shear loading were first considered by Zhou and Wang (2004, 2006) using the Schmidt method. The fracture analyses of FGMEE materials are mostly restricted to a relatively simple anti-plane problems (Feng and Su, 2006; Ma et al., 2007, 2009; Li and Lee, 2008; Lee and Ma, 2010; Rangelov et al., 2011). Up to recent years, there are few research papers (Zhou and Chen, 2008; Ma and Lee, 2009; Rekik et al., 2012; Zhong and Lee, 2012) on the in-plane fracture problems of FGMEE materials.

Theoretical studies are mostly under some rigorous assumptions and thus, lots of actual problems need to be solved by using numerical methods. On the numerical side, except the finite element method (FEM) (Rao and Kuna, 2008b), the boundary element method (BEM), the Meshless local Petrov–Galerkin (MLPG) method and the extended finite element method (XFEM) are mostly used to analyze the fracture problems of homogeneous MEE and FGMEE materials. Garcia-Sanchez et al. (2007), Dong et al. (2008), Rojas-Diaz et al. (2012) and Pasternak (2012) adopted the BEM to investigate the static crack problems of MEE materials. Rojas-Diaz et al. (2010) and Wunsche et al. (2012) employed the BEM to study fracture problems of MEE materials under dynamic loading. Sladek et al. (2008) and Li et al. (2009) applied the MLPG method to analyze a crack in homogeneous MEE media. Sladek et al. (2010, 2011) extended the MLPG method to examine crack problems of FGMEE materials subjected to the stationary and transient thermal and mechanical loading. Rojas-Diaz et al. (2011) and Bhargava and Sharma (2012) used the XFEM in static fracture and quasi-static crack propagation analyses of MEE solids.

The intensity factors (IFs) including stress intensity factors (SIFs), electric displacement intensity factor (EDIF) and magnetic induction intensity factor (MIIF) are the key fracture parameters characterizing the crack-tip fields of linear MEE materials. As a powerful tool solving the fracture parameters, conservation integrals such as the J-integral, the J_i-integral and the M-integral are widely used to study the crack behaviors in pure elastic media in the past decades. Recently, these conservation integrals have also been developed to deal with MEE materials. Wang and Mai (2003) first derived a path-independent J-integral for homogeneous MEE materials. Tian and Rajapakse (2005) discussed the J_i-integral and M-integral for a single crack and multi-crack problems in MEE media. For dynamic fracture problems of MEE solids, Chen (2009) established a dynamic contour integral which is equivalent to the dynamic energy release rate. He pointed out that the dynamic contour integral is path-independent for steady-state crack propagation in the absence of mechanical body force, thermal effect and electricity conduction. In order to decouple modes I and II SIFs in mixed-mode fracture, Stern et al. (1976) proposed the interaction integral for pure elastic solids on the basis of the J-integral by considering two admissible states. Enderlein et al. (2005) developed the interaction integral to study the fracture problems of homogeneous PE materials. Soon later, Rao and Kuna (2008a,b) exploited the interaction integral method for solving the IFs of functionally graded PE and FGMEE media. Due to the convenience in the post-processing of most numerical implementations, such as in FEM and XFEM, the domain form of an integral is generally adopted to replace the contour form. By selecting three types of the auxiliary fields for non-homogenous MEE materials, Rao and Kuna (2008b) gave three corresponding domain formulations of the interaction integral and discussed their precision differences. Recently, the domain form of the interaction integral is widely used in the static crack and quasi-static crack propagation analyses of MEE materials (Rojas-Diaz et al., 2011; Bhargava and Sharma, 2012).

To the best knowledge of the authors, almost all the previous fracture studies are focused on the MEE materials with continuous and differentiable properties and correspondingly, all the interaction integral published previously require material properties to be differentiable. However, most of the MEE materials are typical composites composed of PE and PM phases and therefore, there exist unavoidably material interfaces between different phases. In addition, FGMEE materials actually are at least two-phase particulate composites synthesized in such a way that the volume fractions of the constituent materials vary continuously along a spatial direction to give a predetermined composition profile resulting in a relatively smooth variation of the mechanical properties (Rekik et al., 2012). Experimental studies (Cannillo et al., 2006) show that the microstructure and the interfaces between the constituents affect the fracture behaviors of FGMs obviously and therefore, as the research scale decreases down to a certain level, the interfaces in FGMs have to be considered. In order to analyze MEE materials with complex interfaces effectively, this paper aims to establish a fracture mechanics method which is not require material properties to be continuous and differentiable.

In the previous studies on pure elastic and PE media (Yu et al., 2009, 2010a,b, 2012), the authors have established an interaction integral which is domain-independent for material interfaces. In this paper, the authors will attempt to establish a domain-independent interaction integral for MEE media. Our contributions can be stressed as follows. (1) The interaction integral derived here is domain-independent for material interfaces. Therefore, the present interaction integral method may become an extremely promising technique in the fracture analysis of MEE materials with complex interfaces. (2) The expression of the present interaction integral does not contain any derivatives of MEE properties, which gets rid of the requirement on the differentiability of material properties and thus, facilitates the practical implementation of numerical computations since the derivatives of actual material properties are usually extremely difficult to acquire.

The outline of this paper is as follows. Section 2 reviews the basic equations of MEE materials briefly and introduces an expanded tensor notation to simplify their expressions. Section 3 gives the definitions of the interaction integral and the auxiliary fields for MEE media, and provides the relation between the interaction integral and the IFs. Section 4 derives a new domain form of the interaction integral for MEE media with continuous properties. Section 5 derives the domain form of the interaction integral for MEE media with discontinuous properties and gives the rigorous proof that an arbitrary interface in the integral domain does not affect the value of the interaction integral. Section 6 describes the extended finite element method (XFEM) briefly and provides the discretization of the interaction integral. Section 7 presents several numerical examples. Finally, Section 8 gives a summary and some conclusions.

2. Basic relations for MEE media

For MEE media, the governing equations and the boundary conditions are given first. Then, we will define the expanded tensors by which the expressions of the basic equations will be simplified.

2.1. Governing equations

The field equations for a linear MEE medium subjected to magneto-electro-mechanical loads in the absence of body forces, concentrated electric charges and concentrated magnetic source are:

- Constitutive equations:

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - e_{lij}E_l - h_{lij}H_l \\ D_i &= e_{ikl}\varepsilon_{kl} + \kappa_{il}E_l + \beta_{il}H_l \\ B_i &= h_{ikl}\varepsilon_{kl} + \beta_{il}E_l + \gamma_{il}H_l \end{aligned} \quad (1)$$