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# Thermo-elastic optimization of material distribution of functionally graded structures by an isogeometrical approach

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ABSTRACT

A new isogeometrical procedure for optimization of material composition of functionally graded structures in thermo-mechanical processes is introduced. The proposed method employs a generalized form of the standard isogeometric analysis method, allowing for gradation of material properties through patches. The variations of material properties are captured in a fully isogeometric formulation using the same NURBS basis functions employed for construction of the geometry and approximation of the solution. Subsequently, the applicates of control points that define the surfaces of volume fractions of the constituents are considered as the design variables and obtained by solving the optimization problem using a mathematical programming algorithm. Some numerical examples under thermal and mechanical loadings are considered to demonstrate the performance and applicability of the proposed method. Comparison of the obtained results with those of the other existing approaches such as finite elements and meshfree methods verifies the presented results. It will be seen that the proposed procedure considerably removes the difficulties of the existing methods and provides a promising tool for material design of functionally graded structures.

## 1. Introduction

Functionally graded materials (FGMs) are considered as one of the modern generation of composite materials. These materials are commonly two-phase particulate composites, e.g. ceramic and metallic alloy phases, microscopically engineered to have a smooth spatial variation of material properties in order to improve the overall performance. This is accomplished by fabricating the composite material to have a gradual variation in the relative volume fractions of the constituent phases and microstructure. FGMs are ideal candidates for applications involving severe thermal gradients, ranging from thermal structures in advanced aircraft and aerospace engines to computer circuit boards (Shiota and Miyamoto, 1997). It is important to note that the performance of FGMs is not merely a function of the properties and relative amounts of their material constituents, but is also directly related to the ability of the designer to utilize the materials in the most optimal fashion (Goupee and Vel, 2007). Accordingly, the optimization of material distribution is a fundamental step in the design of FG components, which demands the accurate simulation of its response to applied complex thermo-mechanical loadings.

### 1.1. Techniques of analysis and optimization of FGMs

With FGMs being used mainly for thermal barrier coatings, many studies have focused on thermo-mechanical behavior of these materials under thermal or combined thermo-mechanical loads. For instance, Lü et al. (2006) presented a two-dimensional (2D) thermo-elasticity solution for functionally graded thick beams. Ding et al. (2007) derived an elasticity solution for plane anisotropic FG beams with elastic compliance parameters being arbitrary functions of the thickness coordinate. A semi-analytical elasticity solution for bending and thermal deformations of bi-directional FG beams was introduced by Lü et al. (2008). Also, many researchers investigated thermal fracture of functionally graded plates and shells under thermal loadings or thermal shocks (Feng and Jin, 2009; Guo and Noda, 2010; Guo et al., 2008; Sheng and Wang, 2011; Ueda, 2001). Since, in general, it is not simple to obtain an analytical solution to the partial differential equations with variable coefficients governing to thermo-mechanical response of FGMs, we are usually obliged to resort to numerical methods.

By now, different numerical techniques such as finite difference, finite elements and meshfree methods have been employed for their thermo-mechanical analysis. For instance, Ching and Yen (2005) employed the meshless local Petrov–Galerkin (MLPG) method for the analysis of 2D FG elastic solids under mechanical

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and thermal loads. Wang and Qin (2008) developed a meshless algorithm to simulate the static thermal stress distribution in 2D FGMs on the basis of analog equation theory and the method of fundamental solutions coupling with radial basis functions. Also, Chareonsuk and Vessakosol (2011) investigated numerical solutions for functionally graded solids under thermal and mechanical loads using a high-order control volume finite element method.

On the other hand, lots of strategies have been proposed for the optimization of material distribution of FGMs, thus far. Cho and Ha (2002a) deals with the volume fraction optimization for minimizing steady-state thermal stresses in Ni–$Al_2O_3$ heat-resisting FGM composites where the interior penalty-function and golden section methods are employed as optimization techniques. He also employs the finite difference method for sensitivity analysis and an appropriate material-property estimate for calculating thermo-mechanical properties of the graded layer. Cho and Ha (2002b) address a 2D volume fraction optimization procedure for relaxing the effective thermal stress distribution by using bilinear finite elements for the approximation of volume fraction field. Turteltaub (2002) deals with the control and/or optimization of a two-phase isotropic composite under time-dependent thermo-mechanical loadings. The research concludes that for the minimum structural compliance problem, the optimal distribution of material properties depends on the loading history, even though the deformations are elastic.

In one of the most related numerical works, Goupee and Vel (2006b) investigate the application of the element-free Galerkin method for optimization of material composition of two phase metal-ceramic FGMs in thermo-elasticity problems. The spatial distribution of ceramic volume fraction is obtained by piecewise bicubic interpolation of volume fractions defined at a finite number of grid points. Subsequently a real-coded genetic algorithm is adopted to minimize the peak thermal stress or mass of FG structures. Optimization of volume fractions for functionally graded plates and panels considering stress and critical temperature is investigated in Na and Kim (2009a,b, 2010). Nemat-Alla (2009) discussed reduction of thermal stresses by composition optimization of 2D FGMs assuming analytical power law functions for variations of volume fractions of the constituents. Optimization of material composition of FGMs based on multiscale thermo-elastic analysis for thermal stress relaxation was conducted by Chiba and Sugano (2012). In the presented method, location-dependent unit cells representing the microstructures of two-phase FGMs are created using morphology description functions, and the homogenized material properties and microscale thermal stresses are computed using the asymptotic expansion homogenization method. Recently, Kou et al. (2012) have presented a procedural model for optimal design of FGMs using the particle swarm optimization method. In this work, instead of using the widely used explicit functional models, a feature tree based model is proposed to represent generic material heterogeneities. A procedural model of this sort allows more than one explicit function to be incorporated to describe versatile material gradations and so the material composition at a given location is no longer computed by simple evaluation of an analytic function, but obtained by execution of customizable procedures (Kou et al., 2012). Surendranath et al. (2003) proposed a methodology to enhance the optimization of material distributions in composite structures using gradient architectures. It is demonstrated that genetic algorithms (GAs) can be enhanced for composite structures by constraining the design search space through a reduction in the number of design variables thereby the computational effort is substantially reduced. Some researchers deal with thermo-elastic optimization of FGMs with temperature dependent material properties (Bobaru, 2007; Boussaa, 2009; Goupee and Vel, 2007). Simultaneous optimization of material properties and topology of FG structures was also undertaken in many researches (Almeida et al., 2010; Paulino and Silva, 2005; Xia and Wang, 2008).

It needs to be mentioned that, a developed optimization method for material design of FGMs requires some significant characteristics to be considered as an efficient one. First, the ability of the method in creating complex material profiles using least possible numbers of design variables is definitely of primary importance. This makes the procedure to become computationally more efficient. Next, the smoothness of the obtained optimal material profiles that do not contain discontinuities or jumps throughout the domain is very desirable which can also facilitate their fabrication (Goupee and Vel, 2006a,b). It is evident that the smoothness of the optimal material profile is directly related to the order of continuity and differentiability of the employed material profile. However, considering the literature, one can see that most of reported strategies for material design and optimization of FGMs suffer from major difficulties in these senses.

Regarding the employed design parameterization scheme, we can classify the reported approaches into two main classes of functional models and discrete models. Functional models employ typical analytical functions for description of material gradation through the computational domain so that some coefficients of the functions are selected as the design variables for optimization of the material distribution. It is evident that adopting such a methodology, despite being computationally efficient as well as having higher order of continuity and ease of implementation, lacks the primary advantage of creating complicated material profiles.

On the other hand, discrete models which are more commonly adopted, partition the domain of interest into a collection of subdomains wherein the material properties are either assumed homogeneous or are interpolated. Due to the fact that these models naturally use a larger number of design variables, they are more capable to produce complex material distributions. However, most of them suffer from drawbacks of impossibility of having any desirable order of material distribution continuity throughout the domain as well as demanding a large number of design variables for creation of complex material profiles which makes them computationally inefficient.

A commonly used nodal based approach is the so-called continuous approximation of material distribution (CAMD) technique (Kumar and Gossard, 1996; Matsui and Terada, 2004) which employs the shape functions within the elements and subsequently throughout the design domain to obtain the densities for layout and topology optimization (Almeida et al., 2010). The main idea of this concept was initially introduced by Kim and Paulino (2002) as the generalized graded finite elements for the analysis of heterogeneous materials.

It is a well-known fact that the use of higher-order finite elements for approximation of material distributions is effective against checker-board patterns and results in clearer topologies (Matsui and Terada, 2004). However, we note that higher order shape functions are not always positive within an element. This means that the approximated value of the design variable by these shape functions can be negative locally in an element and violate the geometrical constraint for the design variable, i.e. $0 \leqslant \rho(\mathbf{x}) \leqslant 1$ , even if the nodal design variables are all positive (Matsui and Terada, 2004). Hence, most of researchers have employed linear shape functions for capturing the material gradations inside elements in the graded finite element context in optimization problems, even though the solution is approximated by higher order shape functions (Almeida et al., 2010; Cho and Ha, 2002b). For more details, the interested reader is referred to (Almeida et al., 2010; Matsui and Terada, 2004). It is evident that such a remedy reduces the order of continuity of the material description model and accordingly the smoothness and applicability of the obtained optimal material profile.