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Fracture toughness of open-cell Kelvin foam

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ABSTRACT

Brittle fracture behavior of a perfect open-cell Kelvin foam is considered. The foam is modeled as a spatial lattice consisting of brittle elastic struts rigidly connected to each other at the nodal points. The fracture toughness is determined from the analysis of a quasi-plane problem for a slice of the foam with an embedded finite length crack generated by broken struts. The crack plane is chosen on the basis of a previous study of crack nucleation phenomenon, and the crack length, which assures the self-similar K-field in the tip vicinity, is established by numerical experiments. For the considered densities range the crack includes several hundreds of broken struts and, consequently, the portion of the foam to be considered in the analysis has a very large number of nodal degrees of freedom. The computational cost is reduced significantly by using for the analysis the representative cell method based on the discrete Fourier transform. As a result, the initial problem for the foam slice is reduced to the problem for the repetitive cell which includes 12 struts.

The dependence of the Mode I and Mode II fracture toughness of the considered bending dominated foam upon its relative density is determined and found to be different from known results for the stretch dominated cubic cell lattice. On the other hand, the results obtained for Mode I meet the experimental data and theoretical predictions for random foams. For the case of struts with hollow cross-section the analysis predicts linear dependence of the fracture toughness upon cross-section gyration radius.

1. Introduction

The increasing use of cellular low-density materials in engineering practice stimulated experimental and theoretical studies of their mechanical behavior in recent decades. Their microstructure is generated by a system of separated (for closed-cell material) or interconnected (for open-cell material) voids. In the latter case, the bulk material is concentrated within the struts connecting the nodal points thus creating a lattice-type structure. The variety of these structures can be divided into bending-dominated cellular foams and stretch-dominated lattice trusses (Ashby, 2006; Deshpande et al., 2001).

The effective elastic properties of foams and their compressive strength are well documented. In early theoretical models foam microstructure was represented by perfectly periodic space-filling polyhedral cells: Ko (1965) employed rhombic dodecahedron and the trapezo-rhombic dodecahedron, and most other authors (e.g., Warren and Kraynik (1997), Zhu et al. (1997), Li et al. (2003), Dementev and Tarakanova (1970) and D'Angelo et al. (2012)) used tetrakaidecahedron corresponding to the open-cell Kelvin foam. More accurate and, consequently, more complicated models of real disordered foams are based on employing random microstructures (e.g., Makiyama et al. (2002), Jang et al. (2010), Mangipudi and Onck (2011) and Gaitanaros et al. (2012)). Note, that as shown by Jang and Kyriakides (2009), in many cases the Kelvin foam based models provide a reasonably good approximation.

Less attention has been directed to the modeling of brittle fracture of foams caused by a macrocrack propagation. As for the effective properties, the natural way for such modeling is to start with periodic microstructures and then to address more realistic disordered ones. However, contrary to the case of two-dimensional cellular materials, for which theoretical results for different microstructures have been obtained (e.g., Fleck and Qiu (2007)), modeling spatial foam microstructure with embedded crack is a heavy task even if perfect periodicity is adopted. Therefore the dependence of the fracture toughness upon the foam relative density was determined theoretically only for microstructures with relatively simple geometry. Romijn and Fleck (2007) and Choi and Sankar (2003, 2005) obtained the results for the case of Mode I and Mode II cracks in a cubic lattice, Ryvkin (2012) derived an analytical expression for the case of a Mode III crack in a lattice with rectangular prismatic cells. Kelvin foam geometry was considered by Thiyagasundaram et al. (2011) who found the fracture toughness for a specific value of the relative density. The main goal of the present paper is to carry out a parametric study of the fracture toughness behavior of the open cell Kelvin foam. It would be of interest to compare the obtained theoretical results with

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experiments and with the formula in Ashby (1983) relating the fracture toughness of a foam K_{IC} with its relative density ρ, tensile strength of solid material σ_f and strut length l

$$K_{IC} = \sigma_f C\sqrt{\pi l}\rho^{3/2}. \tag{1}$$

Recall that power 3/2 in this formula is obtained from analytical considerations and the value of coefficient $C = 0.65$ is established by fitting the experimental data for rigid polymeric foams. It should be noted that besides the ability of Kelvin foam to represent actual disordered ones there is an additional reason to address this microstructure. Indeed, recent advances in the printed materials technology allow to produce perfectly periodic brittle ceramics of arbitrary and, in particular, of a tetrakaidecahedron based geometry (Ortona et al., 2012).

The paper is organized as following. In Section 2 the adopted approach to the fracture toughnesses evaluation is formulated and in the next Section the problem of a finite length crack in a lattice produced by the failed struts is presented. The failed struts are modeled by fictitious forces and in Section 4 the solution of the auxiliary problem of a unit force system applied to the pristine lattice is given. In Section 5 the crack length required for the fracture toughness analysis is found, the dependence of the Mode I and Mode II fracture toughness upon the foam microstructure parameters is established in Section 6. Finally, in the last section some conclusions are drawn.

2. Problem statement

The critical value of the stress intensity factor defining the fracture toughness of brittle cellular material is found from the critical stress state of a strut in the crack tip vicinity. Namely, it is assumed that the strut fails when its skin stress reaches the tensile strength of the parent material

$$\sigma_{max} = \sigma_f. \tag{2}$$

The corresponding boundary value problem can be formulated in two ways. The first one is to apply at the remote boundary the displacements corresponding to the K-field for a crack in homogeneous material possessing the effective elastic properties of the cellular body (e.g., Fleck and Qiu (2007) and Thiyagasundaram et al. (2011)). The alternative way is the analysis of a finite length crack embedded into cellular material subjected to a uniform stress at the boundaries (Huang and Gibson, 1991). The importance of considering a material domain of a sufficiently large size in the framework of this approach in order to determine the fracture toughness was emphasized by Quintana and Fleck (2007). An efficient way to meet this requirement is to take advantage of the material microstructure periodicity through application of the discrete Fourier transform – based representative cell method (Ryvkin and Nuller, 1997; Ryvkin, 2008). Theoretically, this method allows to consider an unbounded infinite material domain, however, its implementation (Lipperman et al., 2007a) revealed some numerical problems at the stage of the inverse transform integration. This difficulty is circumvented by considering a sufficiently large finite domain instead of an infinite one and, consequently, by using the finite discrete transform where the inverse transform formula is a sum instead of an integral. This approach adopted also in the present study was successfully applied for the analysis of the local effects in continuous (Ryvkin and Aboudi, 2008; Ryvkin and Aboudi, 2011) and discrete (Kucherov and Ryvkin, 2012) periodic elastic systems.

Consider an open-cell Kelvin foam modeled by a beam lattice consisting of straight-linear struts of length l rigidly connected to each other at the nodal points. A model of this foam produced by 3D printing technology is shown in Fig. 1.[1] The struts have a circular cross-section and are made of an isotropic elastic material with Young modulus E and Poisson ratio ν. They can undergo deformations of stretching, bending and torsion. The corresponding cross-sectional parameters are area A, moment of inertia I and polar moment of inertia J_p. Square faces of the lattice are assumed to be parallel to the coordinate planes of a global Cartesian system (XYZ); consequently, hexagonal faces belong to octahedral planes. It will be convenient to use the crystallographic notation in terms of which the considered microstructure forms a bcc lattice with square faces in $\{100\}$ planes and hexagonal ones in $\{111\}$ planes.

In contrast to homogeneous materials, a crack in a periodic cellular material can propagate in several specific planes defined by the material microstructure. For two-dimensional cellular materials this phenomenon was observed by Lipperman et al. (2007a) and the location of most dangerous planes for the Kelvin foam considered in the present study was determined by Kucherov and Ryvkin (2012). It has been found that independently of the applied tensile loading direction, in a broad range of material densities the crack-like flaw appears at the crystallographic planes $\{101\}$ only. Therefore, in the present paper the fracture properties for this type of planes are examined, and a crack in the plane $(0\bar{1}1)$ is considered (see Fig. 2). The crack of length $2a$ is formed by a number of broken struts normal to this plane, and its front is parallel to X-axis. Consequently, in the case of uniform remote loading the problem possesses translational symmetry defined by the lattice vector $\mathbf{e} = \{2\sqrt{2}l, 0, 0\}$. In addition, since plane strain deformation with $\varepsilon_x = 0$ is considered, the planes which are perpendicular to the X-axis and pass through the square faces are the planes of mirror symmetry. Therefore, the crack problem can be formulated for a slice of foam $\sqrt{2}l$ thick bordered by a pair of such planes indicated in Fig. 2 by dashed lines. The nodes located in these planes have zero displacement in X-direction and zero rotations about the Y and Z axes.

If the slice undergoes Mode I (Mode II) deformation, then for a crack of sufficiently large length

$$a \gg l \tag{3}$$

embedded in a slice of sufficiently large size L

$$L \gg a \tag{4}$$

the fracture toughness can be found from the formula (Ashby, 1983)

$$K_{IC(IIC)} = \sigma_c\sqrt{\pi a}. \tag{5}$$

where σ_c is the critical value of the applied uniform remote tensile (shear) stress under which the strut failure condition (2) is fulfilled.

3. Beam lattice with failed struts

A crack in a cellular material is a plane flaw produced by a number of broken elements. Thus, using a beam lattice model one has to obtain the solution for the portion of a lattice with several removed struts. The volume of required calculations may be reduced significantly by using the finite discrete Fourier transform. The stipulations for its application are the microstructure periodicity, which allows to view the lattice as an assemblage of identical cells, and the continuity of the forces and the displacements at the interfaces between the neighboring cells. The violation of these conditions is circumvented by presenting the solution as a superposition of two solutions for the undamaged lattice. The first one is the periodic solution for the lattice subjected to the remote

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[1] The sample was fabricated from Objet VeroGray and TangoBlackPlus materials using the CONNEX500 3D Printing System, both of Stratasys Ltd. (Rehovot, Israel).