International Journal of Solids and Structures 51 (2014) 478–490

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

On the local nature of the strain field calculation method for measuring heterogeneous deformation of cellular materials

Shenfei Liao [a,b], Zhijun Zheng [a,*], Jilin Yu [a]

[a] CAS Key Laboratory of Mechanical Behavior and Design of Materials, University of Science and Technology of China, Hefei, Anhui 230026, PR China
[b] National Key Laboratory of Shock Wave and Detonation Physics, Institute of Fluid Physics, CAEP, Mianyang, Sichuan 621900, PR China

ARTICLE INFO

Article history:
Received 3 December 2012
Received in revised form 23 September 2013
Available online 23 October 2013

Keywords:
Cellular material
Heterogeneous deformation
Local strain
Cut-off radius
Shock wave

ABSTRACT

A strain field calculation method based on the optimal local deformation gradient technique has been developed to calculate the 'local' strain tensor of cellular materials using cell-based finite element models. The local nature and accuracy of this method may be strongly dependent on the cut-off radius, which is introduced to collect the effective nodes for determining the optimal local deformation gradient of a node. Two different schemes are first analyzed to determine the suitable cut-off radius by characterizing the heterogeneous deformation of Voronoi honeycombs under uniaxial compression and we suggest that in Scheme 1, the cut-off radius defined based on the reference configuration is about 1.5 times the average cell radius; in Scheme 2, the cut-off radius defined based on the current configuration is about 0.5 times the average cell radius. Then, Scheme 3, a combined scheme of the two former schemes, is further suggested. It is demonstrated that the optimal cut-off radius in Scheme 3 characterizes the local strain reasonable well whether the compression rate is low or high. Finally, the strain field calculation method with the optimal cut-off radius is applied to reveal the evolution of the heterogeneous deformation of two different configurations of double-layer cellular cladding under a linear decaying blast load. The 2D fields and the 1D distributions of local engineering strain are calculated. These results interpret the shock wave propagation mechanisms in both claddings and provide useful understanding in the design of a double-layer cellular cladding.

1. Introduction

Man-made cellular materials, such as metallic foams and honeycombs, have been widely used in lightweight structural and multifunctional applications (Gibson and Ashby, 1997). There exist at least two length scales in the characterization of a cellular material due to its cellular nature: the mesoscopic scale (cell level) and the macroscopic scale (specimen level). At the mesoscopic scale, the individual response of a cell strongly depends on its morphology and location in the material. For example, three possible deformation mechanisms at the cell level were revealed in Bastawros et al. (2000). Distortions and rotations arising at multiple cells develop the localized deformation bands of approximately one-cell width, which have been observed in cellular materials under both quasi-static compression (Bastawros et al., 2000; Bastawros and Evans, 2000; Jang and Kyriakides, 2009; Tan et al., 2002) and dynamic compression (Lee et al., 2006; Radford et al., 2005; Tan et al., 2012; Zou et al., 2009). At the macroscopic scale, the response of the cellular material is an average of the responses of cells at the mesoscopic scale. A continuum concept, nominal engineering stress–strain relationship, is commonly used to characterize the macroscopic response of cellular materials without considering the cellular nature. However, deformation localization is a typical feature of the response of cellular materials, especially in dynamic cases, and obviously it cannot be represented by nominal stress–strain relationship (Liu et al., 2009). Due to the cellular nature, the concept of 'local' strain should be introduced to characterize the deformation of cells at the mesoscopic scale. Here, the terminology of 'local' refers to a small region but not to a mathematical point. Zheng et al. (2012) suggested that the local strain in cellular materials should be defined as statistical average measured over the range of at least one cell size. Following this guide, we aim to evaluate the local nature and accuracy of a local strain in a cellular material in this study.

Digital image correlation (DIC) technique based on camera system and image processing system has been used in a considerable number of experimental studies on cellular materials to capture the heterogeneous deformation at the mesoscopic scale. For example, this technique was employed by Bastawros et al. (2000) to monitor the evolution of plastic deformation in a closed-cell aluminum foam under quasi-static compression: at the onset of nonlinear response, localized deformation bands initiate and have width of approximately one-cell diameter; on further straining to plateau

* Corresponding author. Tel.: +86 551 6360 3044; fax: +86 551 6360 6459.
E-mail addresses: zjzheng@ustc.edu.cn, zjzheng@ustc.edu (Z. Zheng).

0020-7683/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijsolstr.2013.10.019

regime, deformation bands contact with each other and develop a characteristic spacing of 3–4 cell diameters. Local deformation in ultra-light open-cell foams (Wang and Cuitio, 2002) and honeycombs (Khan et al., 2012) under quasi-static compression was also revealed by using the DIC technique. Besides the applications in quasi-static loading case, the DIC technique was used by Elnasri et al. (2007) to perform a quantitative measurement of the strain fields during the crushing of metallic cellular structures under impact loading, by means of which shock front propagation was observed and shock front speed was measured through a simplified analysis.

Some numerical methods for characterizing the local strain in cellular materials have been proposed in the literature. For regular honeycombs, Zou et al. (2009) presented a definition of local engineering strain based on relative displacement between two neighboring cross-sections. From the strain distribution, shock front propagating through the honeycomb was captured. Nevertheless, the strain behind the shock front suffers high fluctuations and thus is difficult for quantitative analysis. Besides, this definition of local strain masks the gross behavior over the transverse direction, recognizing the limitation of being more suitable for a high impact velocity. For irregular cellular materials, a local strain map algorithm, similar to the DIC technique, was developed by Mangipudi and Onck (2011) for characterizing the strain fields at the mesoscopic scale. In their work, Voronoi honeycomb cells were triangulated into triangles, in which local strain was assumed to be constant and calculated with displacements of cell nodes by using standard finite element method. The local strain was defined by Cauchy strain formula, which is only appropriate for small-strain states (Mangipudi and Onck, 2011). Recently, Liao et al. (2013a) developed a strain field calculation method based on the optimal local deformation gradient technique for cellular materials. In this approach, Voronoi honeycomb cells in the numerical simulation were discretized into a series of nodes and discrete local deformation gradients at representative nodes, in a least squares sense, were calculated by the relative motions of the representative nodes and their neighboring nodes. Then, a local strain tensor related to the optimal local deformation gradient can be calculated through the continuum mechanics theory and it allows considering general finite-strain states of both regular and irregular cellular materials.

The cut-off radius, which is used to determine the neighboring nodes of a representative node for calculating the optimal local deformation gradient, is a key parameter in the strain field calculation method presented in our previous work (Liao et al., 2013a). The local nature and accuracy of a local strain may be strongly dependent on the cut-off radius for a cellular material. Thus, we aim to extend our previous work (Liao et al., 2013a) by investigating the effect of the cut-off radius on the local strain and to find an optimal choice of the cut-off radius. A brief introduction of the formulation of the local strain tensor with different schemes for the definition of the cut-off radius is presented in Section 2. The effect of the cut-off radius on the local nature and accuracy of the local strain is analyzed and an optimal choice of the cut-off radius is suggested in Section 3. As an application of the strain field calculation method, heterogeneous deformations of Voronoi honeycomb cores in two different configurations of double-layer cellular cladding under blast load are characterized in Section 4, followed by conclusions in Section 5.

2. Formulation of local strain

2.1. Local strain tensor

According to the continuum mechanics theory, large deformation can be represented by the Lagrangian or Green strain tensor, **E**, given by

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^{\mathrm{T}} \cdot \mathbf{F} - \mathbf{I}\right), \tag{1}$$

where **F** is the deformation gradient, **I** the identity matrix and superscript T denotes the transpose of a matrix. Once a local deformation gradient is determined, a local strain tensor is obtained. However, due to the cellular nature, the deformation gradient cannot be directly constructed in a cellular material. Therefore, a discrete local deformation gradient determined by the method of least squares defined in Li and Shimizu (2005), Gullett et al. (2008) and Zimmerman et al. (2009) was introduced in our previous work (Liao et al., 2013a) to develop a strain field calculation method for characterizing the local deformation of a cellular material. To perform the strain field calculation, the locations of nodes are monitored and output from the cell-based finite element model. For example, a Voronoi honeycomb is discretized into a series of corner nodes (vertices of Voronoi honeycomb cells) and other nodes on the cell walls, as illustrated in Fig. 1. The deformation of the Voronoi honeycomb is reflected by the relative motions of all nodes. The interior strain of the Voronoi honeycomb is then assumed to be discretely represented by the local strains at all nodes. For the purpose of saving computational cost, we sample the local strains at corner nodes rather than at all nodes to discretely represent the interior strain of a Voronoi honeycomb in the present work. The reasonableness of this simplification will be evaluated later.

Two nodal configurations, namely the reference (undeformed) configuration Ω_0 and the current (deformed) configuration Ω_1, are needed to calculated the local deformation gradient. For node i and its neighboring node j, their relative position vectors are $\mathbf{U}_{ij} = \mathbf{X}_j - \mathbf{X}_i$ and $\mathbf{u}_{ij} = \mathbf{x}_j - \mathbf{x}_i$ in configurations Ω_0 and Ω_1, respectively, where **X** and **x** are the position vectors of a node in Ω_0 and Ω_1, respectively, as illustrated in Fig. 1. They are all considered to be *column* vectors. It is assumed that there exists an optimal local deformation gradient $\mathbf{F}_i$ defined at node i, which best maps node i and all of its neighboring nodes from Ω_0 to Ω_1 by $\mathbf{u}_{ij} \approx \mathbf{F}_i \cdot \mathbf{U}_{ij}$. In other words, the optimal local deformation gradient $\mathbf{F}_i$ minimizes the least squares mapping error of node i defined as (Li and Shimizu, 2005)

$$\varphi_i = \sum_{j=1}^{N} (\mathbf{F}_i \cdot \mathbf{U}_{ij} - \mathbf{u}_{ij})^{\mathrm{T}} \cdot (\mathbf{F}_i \cdot \mathbf{U}_{ij} - \mathbf{u}_{ij}), \tag{2}$$

where N is the number of neighboring nodes of node i. To ensure the local nature of the local strain, a cut-off radius R_c is introduced to determine the neighboring nodes of node i and will be discussed later. Based on the method of least squares, the optimal local deformation gradient $\mathbf{F}_i$ is determined by

$$\frac{\partial \varphi_i}{\partial \mathbf{F}_i} = 2\sum_{j=1}^{N} (\mathbf{F}_i \cdot \mathbf{U}_{ij} - \mathbf{u}_{ij}) \cdot \mathbf{U}_{ij}^{\mathrm{T}} = 0. \tag{3}$$

The solution of Eq. (3) is given by Li and Shimizu (2005)

$$\mathbf{F}_i = \mathbf{W}_i \cdot \mathbf{V}_i^{-1}, \tag{4}$$

where matrix $\mathbf{V}_i$ and $\mathbf{W}_i$ are

$$\mathbf{V}_i = \sum_{j=1}^{N} \mathbf{U}_{ij} \cdot \mathbf{U}_{ij}^{\mathrm{T}}, \quad \mathbf{W}_i = \sum_{j=1}^{N} \mathbf{u}_{ij} \cdot \mathbf{U}_{ij}^{\mathrm{T}}. \tag{5}$$

2.2. Possible choices of the cut-off radius

The choice of neighboring nodes plays an important role in the calculation of the discrete local deformation gradient at a node. Intuitively, neighboring nodes within a certain proximity can