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A variational model for stress analysis in cracked laminates with arbitrary symmetric lay-up under general in-plane loading

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A B S T R A C T

The present research work presents a variational approach for stress analysis in a general symmetric laminate, having a uniform distribution of ply cracks in a single orientation, subject to general in-plane loading. Using the principle of minimum complementary energy, an optimal admissible stress field is derived that satisfies equilibrium, boundary and traction continuity conditions. Natural boundary conditions have been derived from the variational principle to overcome the limitations of the existing methodology on the analysis of general symmetric laminates. Thus, a systematic way to formulate boundary value problem for general symmetric laminates containing many cracked and un-cracked plies has been derived, and appropriate mathematical tools can then be employed to solve them. The obtained results are in excellent agreement with the available results in the literature. In the field of matrix cracks analysis for symmetric laminates, the present formulation is the most complete variational model developed so far.

1. Introduction

In many branches of engineering, like aerospace and civil engineering, composite laminates are increasingly used as structural components. The observed damage process of laminated composites during operation is rather complex consisting of matrix cracks, delaminations, fiber-matrix debondings, fiber breakage, etc. However, matrix cracking parallel to the fiber direction on off-axis plies is usually the first damage mode observed for in-plane loading. Although matrix cracking is not critical from a final fracture point of view, its presence triggers the initiation of other damage modes like delamination and fiber breakage or provides pathways for entry of corrosive liquids that may subsequently lead to fracture. In addition, matrix cracks lead to stiffness reduction and stress redistribution to adjacent plies, which are needed for computation of fiber-dominated failure modes and laminate strength. Moreover, the first step in any matrix cracking analysis is to obtain stress state for laminate containing matrix cracks. Therefore, the capability in analyzing stress field of a cracked laminate has played an important role in the development of damage mechanics for laminated composites. Such capability has evolved over decades to build up in terms of accuracy and versatility, from ply-discount method in early days to shear-lag model (Garrett and Bailey, 1977), stress-based variational approach (Hashin, 1985), finite element (Herakovich et al., 1988), stress transfer model (McCartney, 1992), finite strip method (Li et al., 1994), displacement-based variational approach (Berthelot et al., 1996), etc. The outcome of such micromechanical stress analyses offer supports to other approaches in damage mechanics, such as self-consistent approach (Laws et al., 1983), continuum damage mechanics (Lundmark and Varna, 2005; Singh and Talreja, 2008) and discrete damage mechanic (Barbero et al., 2011). The reader is referred to the following publications for a detailed review of the developments in the area of matrix cracking analysis (Nairn, 2000a; Berthelot, 2003).

Among the approximate analytical models, the stress-based variational approach (Hashin, 1985, 1986, 1987; Nairn, 1989; Nairn and Hu, 1992; Kuriakose and Talreja, 2004; Vinogradov and Hashin, 2010) and stress transfer model (McCartney, 1992, 2000; McCartney and Pierse, 1997; Katerelos et al., 2006) have shown to be more accurate from the micromechanical point of view (Berthelot, 1997; Nairn, 2000a,b) in comparison to other methods (e.g. shear-lag).

Stress transfer model of McCartney (1992) is a 2D analysis, which considers the stress and displacement components based on the generalized plane strain assumptions. The analysis satisfies the equilibrium equations, the interface continuities and the boundary conditions. However, some of the stress-strain relations and boundary conditions are satisfied in an average sense. McCartney (2000) has extended the stress transfer model to analyze general symmetric laminates with arbitrary stacking sequence, having a uniform distribution of ply cracks in a single orientation, under general in-plane loading. The technique is basically analytical,

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but because of the resulting complexity, the analysis must be handled numerically in some steps while making predictions of the behavior of laminate. The stress transfer model can use the ply refinement technique (Takeda et al., 2000; McCartney and Pierse, 1997) where each layer of the laminate is subdivided into plies having the same properties in order that important through the thickness variations of the stress and displacement components could be taken into account. Today, stress transfer model is considered as one of the most efficient, versatile and accurate methods to analyze stress field for the laminate containing matrix cracks.

Hashin (1985) has analyzed the stress distributions in cross ply cracked laminates under tension or shear using a stress-based variational approach. He has presented an approximate 2D stress representation, which automatically satisfies the equilibrium equations, interface and boundary conditions that must be satisfied by stress components. The so-called admissible stress field was then used in conjunction with variational techniques to minimize the complementary energy and thus to provide an optimal solution for the stress field. It has been revealed that the stress field obtained by the variational approach does an excellent job of predicting stiffness reduction and crack grow experiments (Vinogradov and Hashin, 2010; Nairn, 2000a,b). Nevertheless, the variational approach has mostly been used for treatment of either cross-ply laminates (Hashin, 1985, 1986, 1987; Nairn, 1989; Varna and Berglund, 1992; Nairn and Hu, 1992; Rebiere et al., 2001; Kuriakose and Talreja, 2004) or other symmetric laminates that by averaging out the off-axis plies are reduced to cross-ply (Joffe and Varna, 1999; Li and Lim, 2005). Recently, Vinogradov and Hashin (2010) have extended the capability of the variational approach to analyze stress field and consequently stiffness reduction of angle ply laminates. It should be noted that the mathematical model for all mentioned variational works involves effectively only two layers, one cracked and one un-cracked, representing a three-layered laminate after applying symmetry considerations. Therefore, these models do not have the capability of analyzing stress field for the laminates with multiple cracked and un-cracked layers, which cannot be simplified to a two-layer model using symmetry, due to the lack of boundary conditions for un-cracked layers. More recently, Li and Hafeez (2009) have overcome this drawback by introducing some boundary conditions as an outcome of variational procedure and translational symmetry (Li et al., 2009), called natural boundary conditions, in the terminology of variational calculus. As a result, the applicability of the variational approach has been extended fundamentally for considering multiple layers laminates. However, their model (Li and Hafeez, 2009) has only considered cross ply laminates under axial loading due to the assumed admissible stress field.

In the current work, an attempt has been made to extend the applicability of the variational approach for analyzing cracked symmetric laminates with arbitrary stacking sequence under general in-plane loading. An admissible stress field that satisfies equilibrium and all the boundary and continuity conditions is constructed and it is used in conjunction with the principle of minimum complementary energy to achieve the optimal stress state of a general symmetric cracked laminate with multiple cracked and un-cracked layers. A systematic way of evaluating governing equations is developed, which is completely analytical, and consequently, the model could enjoy the advantages of ply refinement technique. To donduct the analysis for considering general symmetric lay-ups, a set of natural boundary conditions is considered as an outcome of variational procedure and translational symmetry. Stress state of cracked laminate estimated by the suggested approach is in excellent agreement with the results obtained from the formulations of Hashin (1985) and Li and Hafeez (2009), which are special cases of the current formulation.

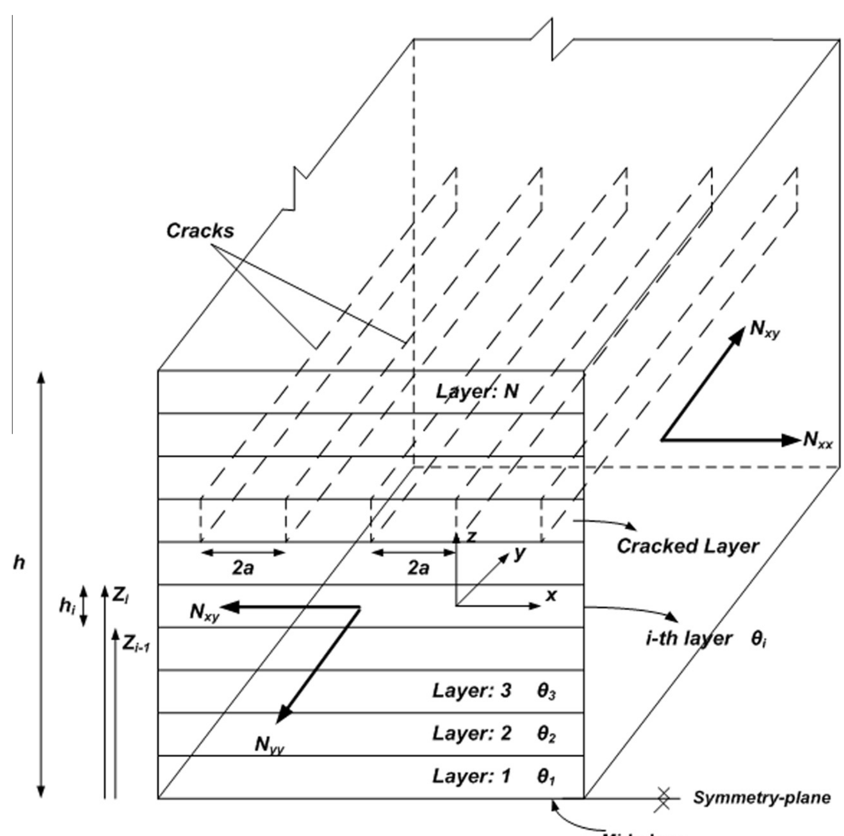

Fig. 1. Geometry of an arbitrary symmetric laminate containing cracks in one 90° layer (only the upper set of N layers ($z > 0$) is shown).