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Proper Generalized Decomposition and layer-wise approach for the modeling of composite plate structures

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ABSTRACT

In the framework of the design of laminated and sandwich structures, the computation of local quantities needs a layer-wise approach. But, the computational cost of such approach increases with the number of layers. In this work, the introduction of the Proper Generalized Decomposition (PGD) is presented for the layer-wise modeling of heterogeneous structures in order to reduce the number of unknowns. The displacement field is approximated as a sum of separated functions of the in-plane coordinates x, y and the transverse coordinate z. This choice yields to an iterative process that consists of solving a 2D and 1D problem successively at each iteration. In the thickness direction, a fourth-order expansion in each layer is considered. For the in-plane description, classical Finite Element method is used.

After a preliminary study to show the relevance of the present approach, mechanical tests for thin to thick laminated and sandwich plates with various boundary conditions are presented. The results are compared with elasticity reference solutions.

1. Introduction

Composite and sandwich structures are widely used in the industrial field due to their excellent mechanical properties, especially their high specific stiffness and strength. In this context, they can be subjected to severe mechanical loads. For composite design, accurate knowledge of displacements and stresses is required. So, it is important to take into account effects of the transverse shear deformation due to the low ratio of transverse shear modulus to axial modulus, or failure due to delamination ... In fact, they can play an important role on the behavior of structures in services, which leads to evaluate precisely their influence on local stress fields in each layer, particularly on the interface between layers.

According to published research, various theories in mechanics for composite or sandwich structures have been developed. On the one hand, the Equivalent Single Layer approach (ESL) in which the number of unknowns is independent of the number of layers, is used. But, the transverse shear and normal stresses continuity on the interfaces between layers are often violated. We can distinguish the classical laminate theory (Tanigawa et al., 1989) (unsuitable for composites and moderately thick plates), the first order shear deformation theory (Yang et al., 1966), and higher order theories with displacement (Cook and Tessler, 1998; Kant and Swaminathan, 2002; Librescu, 1967; Lo et al., 1977; Matsunaga, 2002; Reddy, 1984; Whitney and Sun, 1973; Polit et al., 2012) and mixed (Carrera, 2000; Kim and Cho, 2007) approaches. On the other hand, the layerwise approach (LW) aims at overcoming the restriction of the ESL concerning the discontinuity of out-of-plane stresses on the interface layers and taking into account the specificity of layered structure. But, the number of dofs depends on the number of layers. We can mention the folllowing contributions (Ferreira, 2005; Icardi, 2001; Pagano, 1969; Reddy, 1989; Shimpi and Ainapure, 2001) within a displacement based approach and (Carrera, 1999, 2000; Rao and Desai, 2004) within a mixed formulation. As an alternative, refined models have been developed in order to improve the accuracy of ESL models avoiding the additional computational cost of LW approach. Based on physical considerations and after some algebraic transformations, the number of unknowns becomes independent of the number of layers. Whitney (1969) has extended the work of Ambartsumyan (1969) for symmetric laminated composites with arbitrary orientation and a quadratic variation of the transverse stresses in each layer. So, a family of models, denoted zig-zag models, was derived (see Kapuria et al., 2003; Lee et al., 1992; Sciuva and Icardi, 2001). Note also the refined approach based on the Sinus model (Vidal and Polit, 2008, 2009, 2011). This above literature deals with only some aspects of the broad research activity about models for layered structures and corresponding finite element formulations. An extensive assessment of different approaches has been made in Carrera, 2002, 2003; Noor and Burton, 1990; Reddy, 1997; Zhang and Yang, 2009.

Over the past years, the Proper Generalized Decomposition (PGD) has shown interesting features in the reduction model framework (Ammar et al., 2006). It has been used in the context

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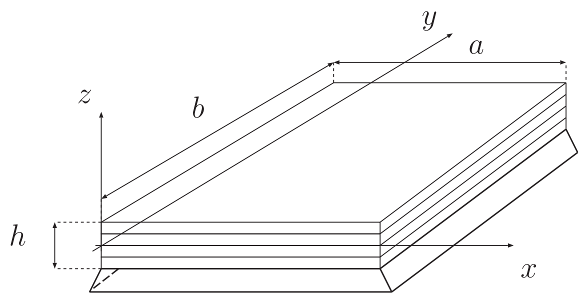

Fig. 1. The laminated plate and coordinate system.

of separation of coordinate variables in multi-dimensional PDEs (Ammar et al., 2006). And in particular, it has been applied for composite plates in Savoia and Reddy (1992) based on Navier-type solution and Bognet et al. (2012) using Finite Element (FE) method.

This work is based on the separation representation where the displacements are written under the form of a sum of products of bidimensional polynomials of (x,y) and unidimensional polynomials of z. In Bognet et al. (2012), a piecewise linear variation in the thickness direction is used, whereas in the present work, a piecewise fourth-order Lagrange polynomial of z is chosen. As far as the variation with respect to the in-plane coordinates is concerned, a 2D eight-node quadrilateral FE is employed. Using the PGD, each unknown function of (x,y) is classically approximated using one degree of freedom (dof) per node of the mesh and the LW unknown functions of z are global for the whole plate. Finally, the deduced non-linear problem implies the resolution of two linear problems alternatively. This process yields to a 2D and a 1D problems in which the number of unknowns is smaller than a classical layerwise approach. The interesting feature of this approach lies on the possibility to have a higher-order z-expansion and to refine the description of the mechanical quantities through the thickness without increasing the computational cost. This is particularly suitable for the modeling of composite structures.

We now outline the remainder of this article. First the mechanical formulation is given. Then, the principles of the PGD are briefly recalled in the framework of our study. The particular assumption on the displacements yields a non-linear problem. An iterative process is chosen to solve this one. The FE discretization is also described. Then, numerical tests are performed. A homogeneous case is first considered to compare the linear approach developed in Bognet et al. (2012) and the present fourth-order expansion with the same number of dofs. Different convergence studies are addressed. Then, the modeling of various laminated and sandwich plates under a global or localized pressure is addressed. The influence of the slenderness ratio is studied. The accuracy of the results is evaluated by comparisons with an exact 3D theory for laminates in bending (Pagano, 1970). A special attention is pointed towards the capacity of the approach to capture local effects and the evaluation of its range of validity. Finally, the computational complexity of the method is given and compared with classical layerwise approach.

2. Reference problem description

2.1. The governing equations

Let us consider a plate occupying the domain $\mathcal{V} = \Omega \times \Omega_z$ with $\Omega_z = [-\frac{h}{2}, \frac{h}{2}]$ in a Cartesian coordinate (x,y,z). The plate is defined by an arbitrary region Ω in the (x,y) plane, located at the midplane for $z = 0$, and by a constant thickness h. See Fig. 1.

2.1.1. Constitutive relation

The plate can be made of NC perfectly bonded orthotropic layers. Using matrix notation, the three dimensional constitutive law of the kth layer is given by:

$$\begin{bmatrix} \sigma_{11}^{(k)} \\ \sigma_{22}^{(k)} \\ \sigma_{33}^{(k)} \\ \sigma_{23}^{(k)} \\ \sigma_{13}^{(k)} \\ \sigma_{12}^{(k)} \end{bmatrix} = \begin{bmatrix} C_{11}^{(k)} & C_{12}^{(k)} & C_{13}^{(k)} & 0 & 0 & C_{16}^{(k)} \\ & C_{22}^{(k)} & C_{23}^{(k)} & 0 & 0 & C_{26}^{(k)} \\ & & C_{33}^{(k)} & 0 & 0 & C_{36}^{(k)} \\ & & & C_{44}^{(k)} & C_{45}^{(k)} & 0 \\ & sym & & & C_{55}^{(k)} & 0 \\ & & & & & C_{66}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_{11}^{(k)} \\ \varepsilon_{22}^{(k)} \\ \varepsilon_{33}^{(k)} \\ \gamma_{23}^{(k)} \\ \gamma_{13}^{(k)} \\ \gamma_{12}^{(k)} \end{bmatrix} \quad \text{i.e. } [\sigma^{(k)}] = [C^{(k)}][\varepsilon^{(k)}] \tag{1}$$

where we denote the stress vector $[\sigma]$, the strain vector $[\varepsilon]$ and $C_{ij}^{(k)}$ the three-dimensional stiffness coefficients of the layer (k).

2.1.2. The weak form of the boundary value problem

Using the above matrix notation and for admissible displacement $\delta\vec{u} \in \delta U$, the variational principle is given by:

find $\vec{u} \in U$ (space of admissible displacements) such that

$$-\int_{\mathcal{V}} [\varepsilon(\delta\vec{u})]^T [\sigma(\vec{u})] d\mathcal{V} + \int_{\mathcal{V}} [\delta u]^T [b] d\mathcal{V} + \int_{\partial\mathcal{V}_F} [\delta u]^T [t] d\partial\mathcal{V} = 0, \quad \forall \delta\vec{u} \in \delta U \tag{2}$$

where $[b]$ and $[t]$ are the prescribed body and surface forces applied on $\partial\mathcal{V}_F$.

3. Application of the Proper Generalized Decomposition to plate

In this section, we briefly introduce the application of the PGD for plate analysis. This work is an extension of the previous studies on beam structures (Vidal et al., 2012a,b).

3.1. The displacement and the strain field

The displacement solution $(u_1(x,y,z), u_2(x,y,z), u_3(x,y,z))$ is constructed as the sum of N products of functions of in-plane coordinates and transverse coordinate ($N \in \mathbb{N}$ is the order of the representation)

$$[u] = \begin{bmatrix} u_1(x,y,z) \\ u_2(x,y,z) \\ u_3(x,y,z) \end{bmatrix} = \sum_{i=1}^{N} \begin{bmatrix} f_1^i(z)\, v_1^i(x,y) \\ f_2^i(z)\, v_2^i(x,y) \\ f_3^i(z)\, v_3^i(x,y) \end{bmatrix} \tag{3}$$

where (f_1^i, f_2^i, f_3^i) are defined in Ω_z and (v_1^i, v_2^i, v_3^i) are defined in Ω. In this paper, a classical eight-node FE approximation is used in Ω and a LW description is chosen in Ω_z as it is particulary suitable for the modeling of composite structure. The strain derived from Eq. (3) is

$$[\varepsilon(u)] = \sum_{i=1}^{N} \begin{bmatrix} f_1^i\, v_{1,1}^i \\ f_2^i\, v_{2,2}^i \\ (f_3^i)'\, v_3^i \\ (f_2^i)'\, v_2^i + f_3^i\, v_{3,2}^i \\ (f_1^i)'\, v_1^i + f_3^i\, v_{3,1}^i \\ f_1^i\, v_{1,2}^i + f_2^i\, v_{2,1}^i \end{bmatrix} \tag{4}$$

where the prime stands for the classical derivative $\left(f_i' = \frac{df_i}{dz}\right)$, and $()_{,\alpha}$ for the partial derivative.