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Flexible bridge decks suspended by cable nets. A constrained form finding approach

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ABSTRACT

The initial geometry of structures made of cables is steered by the cable tensioning forces. In a cable net the geometrical shape and the internal force distribution cannot be dealt as separate issues: the set of geometries defines also the feasible sets of the internal forces. During the last decades, many different approaches have been proposed to deal with the form finding of cable structures. The most efficient one is the so called Force Density Method (FDM), proposed by Schek, which allows to conforming cable nets for structural applications without requiring any further assumption, neither on the geometry, nor on the material properties. An Extension of the Force Density Method, the EFDM, makes it possible to set conditions in terms of fixed nodal reactions or, in other words, to fix the position of a certain number of nodes and, at the same time, to impose the intensity of the reaction forces. Through such an extension the EFDM enables us to deal with form finding problems of cable nets subjected to given constraints and in particular to treat mixed structures, made of cables and struts. In this paper we consider cable nets interacting with members having flexural behaviour. For a given cable assembly and for a given loading condition, aim of this work is to find that particular pretensioning system which replaces both the static and the kinematic functions of the inner reactions of a flexural elastic continuous beam. It is, for instance, the case of the bridge decks suspended by cables, shaped in various forms. The specialization of the EFDM to this type of problem is presented and a progressive set of examples shows the efficiency and the versatility of this approach in contributing to the design of new creative forms.

1. Introduction

The initial geometry of the structures made of cables is steered by the cable tensioning forces. In a cable net the geometrical shape and the internal force distribution cannot be dealt as separate issues, as it happens in the case of the conventional structures: the set of geometries defines also the feasible sets of the internal forces. In the Sixties, when the first lightweight structures of this type were built, the only way to design cable nets was to resort to the use of physical models. The cable net form, the cutting pattern and the behaviour under external load were studied and measured through scale models and then assumed as basis for the design. In the same years the first rational solutions of the form finding problem were introduced. Barnes proposed a dynamical relaxation method (Barnes, 1975). Argyris developed a FEM approach suitable to deal with prestressed cable nets (Argyris et al., 1974). At the beginning of the Seventies, Linkwitz and Schek proposed the so-called Force Density Method (Linkwitz and Schek, 1971; Schek, 1974), that allows to conform cable nets for structural applications without requiring any further assumption, neither on the geometry, nor on the material properties. In its linear version, the final shape is defined through a special parametrization driven by the force densities. Schek presents also a non linear Force Density Method, that allows to deal with constraints concerning imposed relative distances between the nodes, the tensile level in the elements and/or their initial undeformed length. In this approach, the parametrization is not related to the nodes coordinates, but to each truss element. As Descamps et al. (2011) clearly observe, there is no direct control on free nodes coordinates. This is not a limit of the method, since it coherently assumes the nodes as free variable, otherwise the search of the form would be devoid of meaning. However, in dealing with systems in which it is necessary to fix the position of some additional nodes and at the same time to impose the value of the external force (as in the case of structures having a flexible beam/girder suspended to a cable net, or cable struts assemblies), many difficulties arise and the drawbacks of the FDM are self evident. In dealing with cable struts assemblies, Mollaert (1984) suggested an approach where the compression members are replaced by external forces. Tibert (1999) shows the possibility to overcame the drawnbacks by introducing virtual elements in order to satisfy the specific requirements. The use of virtual elements is proposed also in Descamps

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et al. (2011) in dealing with lightweight bridge structures. In this last approach, the force densities in the virtual elements are gradually modified until they reach the fixed node location through an iterative procedure called Constrained Force Density Method. Another work that handles the geometrical position of some nodes of the net is the one proposed by Morterolle et al. (2012), that can be used to calculate geodesic tension trusses that ensure both appropriate node positioning and uniform tension. Others contributions in the force density field come from Miki and Kawaguchi (2010), who reformulate the FDM in terms of functionals, on the basis of variational principle.

In this paper, the aforementioned problems are not solved by introducing virtual elements or virtual forces, but by proposing the missing relation between the force densities and the quantities related to the nodes. This is done through two steps: (1) computation of the reaction forces by using the same matrixes and vectors of the original work proposed by Schek; (2) writing the additional conditions in matricial form. This development can hence be applied in addition to the conditions posed by Schek (initial element lengths, final element lengths, element forces). The paper therefore extends coherently the operational capabilities of the original FDM, that becomes suitable not only to control the quantities related to the elements, but also the ones related to the nodes. The approach proposed can be used both in dealing with cable-struts systems as shown in Malerba et al. (2012) as well as in the case of structures having a flexible beam/girder suspended to a cable net. With this purpose, we consider cable nets interacting with members having flexural behaviour. It is the case of the long span bridges, whose deck girders are suspended at cables or supported by stays (Fig. 1). Whether using suspending cables or curtains of stays, the first design task concerns the setting of the initial configuration, which, for bridges, means the deck girder has to be horizontal or slightly cambered. Due to the interaction with the cables, a new development of the form finding problem is set. In the simplest view, cables or stays supply the static and the kinematic roles of the inner supports of a continuous beam (Fig. 1). The attainment of such a result requires a suitable pretensioning of the suspending system. The pretensioning of the cables is the means used to assign the initial configuration. In the case of stayed structures, for which the tension hardening behaviour of the suspending system is crucial, the pretensioning also provides the cables with the right stiffness and makes them able to play the static role assumed at the basis of these systems. In Sections 1 and 2 the FDM, in its linear and non linear forms, and the EFDM are recalled. Section 3 presents the specialization of the EFDM suitable to determine that particular pretensioning system which replaces the forces at the inner supports of the girder beam. A set of examples will show the efficiency and the accuracy of this approach in dealing with supporting cable curtains lying in a single plane, in two different planes or, generically, in the space. The same examples contribute to show the versatility of the method in helping the design of new original and creative forms.

2. An outline of the Force Density Method

We refer to a cable net and assume that:

– the net is made of straight cable elements, connected at the nodes. Part of the nodes is free, part of them is fixed;
– the net connectivity is known and its geometry is defined by the nodal coordinates;
– the cable elements are weightless;
– the net is subjected to concentrated forces, applied at the nodes.

The net has n free nodes and n_f fixed nodes, connected by m elements. The total number of nodes is $n_s = n + n_f$.

With reference to the ith node of a 3D net (Fig. 2), the equilibrium equations in the x,y,z directions are respectively:

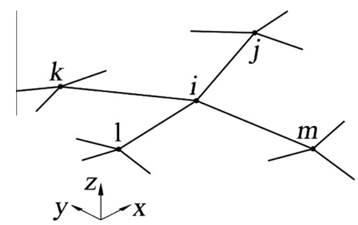


Fig. 2. Generical free node.

$$\begin{cases} T_{ij}\frac{x_j-x_i}{L_{ij}} + T_{ik}\frac{x_k-x_i}{L_{ik}} + T_{il}\frac{x_l-x_i}{L_{il}} + T_{im}\frac{x_m-x_i}{L_{im}} + F_{xi} = 0; \\ T_{ij}\frac{y_j-y_i}{L_{ij}} + T_{ik}\frac{y_k-y_i}{L_{ik}} + T_{il}\frac{y_l-y_i}{L_{il}} + T_{im}\frac{y_m-y_i}{L_{im}} + F_{yi} = 0; \\ T_{ij}\frac{z_j-z_i}{L_{ij}} + T_{ik}\frac{z_k-z_i}{L_{ik}} + T_{il}\frac{z_l-z_i}{L_{il}} + T_{im}\frac{z_m-z_i}{L_{im}} + F_{zi} = 0. \end{cases} \quad (1)$$

where T_{ij} is the axial force and L_{ij} is the length of the element between the nodes i and j:

$$L_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}. \quad (2)$$

2.1. Matrix formulation

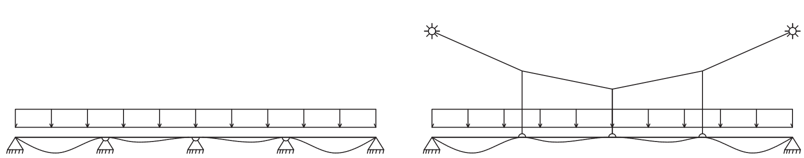
In order to set out the equilibrium equations into a matrix form, the following vectors and matrices are introduced:

- $\mathbf{x}_s, \mathbf{y}_s, \mathbf{z}_s, [n_s \times 1]$, coordinates of the free nodes. By numbering the set of the fixed nodes after that of the free ones, the three vectors can be partitioned into the following subvectors:
  - $\mathbf{x}, \mathbf{y}, \mathbf{z}, [n \times 1]$, coordinates of the free nodes;
  - $\mathbf{x}_f, \mathbf{y}_f, \mathbf{z}_f, [n_f \times 1]$, coordinates of the fixed nodes;
- $\mathbf{f}_x, \mathbf{f}_y, \mathbf{f}_z, [n \times 1]$, nodal forces;
- $\mathbf{l}, [m \times 1]$, length of the elements; $\mathbf{L} = diag(\mathbf{l})$;
- $\mathbf{t}, [m \times 1]$, axial forces in the elements.

We define also a connectivity matrix $\mathbf{C}_s$, having dimensions $[m \times n_s]$, whose terms are:

$$c_s(e) = \begin{cases} +1 & \text{if } i = 1, \\ -1 & \text{if } i = 2, \\ 0 & \text{in the other cases.} \end{cases} \qquad \mathrm{e} = 1, 2, \ldots, \mathrm{m} \quad (3)$$

The difference between the couples of coordinates in the three directions x,y,z, can be written as:



(a) Supported beam. (b) Hanged beam.

Fig. 1. Form finding of a cable suspending a flexible deck.