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Three dimensional analytical solution for finite circular cylinders subjected to indirect tensile test

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ABSTRACT

This paper derives a new three-dimensional (3-D) analytical solution for the indirect tensile tests standardized by ISRM (International Society for Rock Mechanics) for testing rocks, and by ASTM (American Society for Testing and Materials) for testing concretes. The present solution for solid circular cylinders of finite length can be considered as a 3-D counterpart of the classical two dimensional (2-D) solutions by Hertz in 1883 and by Hondros in 1959. The contacts between the two steel diametral loading platens and the curved surfaces of a cylindrical specimen of length H and diameter D are modeled as circular-to-circular Hertz contact and straight-to-circular Hertz contact for ISRM and ASTM standards respectively. The equilibrium equations of the linear elastic circular cylinder of finite length are first uncoupled by using displacement functions, which are then expressed in infinite series of some combinations of Bessel functions, hyperbolic functions, and trigonometric functions. The applied tractions are expanded in Fourier–Bessel series and boundary conditions are used to yield a system of simultaneous equations. For typical rock cylinders of 54 mm diameter subjected to ISRM indirect tensile tests, the contact width is in the order of 2 mm (or a contact angle of 4°) whereas for typical asphalt cylinders of 101.6 mm diameter subjected to ASTM indirect tensile tests the contact width is about 10 mm (or a contact angle of 12°). For such contact conditions, 50 terms in both Fourier and Fourier–Bessel series expansions are found sufficient in yielding converged solutions. The maximum hoop stress is always observed within the central portion on a circular section close to the flat end surfaces. The difference in the maximum hoop stress between the 2-D Hondros solution and the present 3-D solution increases with the aspect ratio H/D as well as Poisson's ratio ν. When contact friction is neglected, the effect of loading platen stiffness on tensile stress in cylinders is found negligible. For the aspect ratio of $H/D = 0.5$ recommended by ISRM and ASTM, the error in tensile strength may be up to 15% for both typical rocks and asphalts, whereas for longer cylinders with H/D up to 2 the error ranges from 15% for highly compressible materials, and to 60% for nearly incompressible materials. The difference in compressive radial stress between the 2-D Hertz solution or 2-D Hondros solution and the present 3-D solution also increases with Poisson's ratio and aspect ratio H/D. In summary, the 2-D solution, in general, underestimates the maximum tensile stress and cannot predict the location of the maximum hoop stress which typically locates close to the end surfaces of the cylinder.

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1. Introduction

One distinct mechanical characteristic of brittle materials is that they are strong in compression but much weaker in tension. Therefore, tensile strength is a very important index in describing brittle materials because it is more relevant to the mechanical failure of brittle solids than compressive strength. However, direct tensile test is very difficult to apply to brittle materials without inducing any eccentric moment. Therefore, various types of indirect tensile tests have been developed in order to measure the tensile strength of brittle materials, including the diametral compression on disk with central hole (Hobbs, 1965), the point load strength test (Wei et al., 1999; Chau and Wei, 2001; Wei and Chau, 2002), the double-punch test (Wei and Chau, 2000), and the diametral compression on the curved surface of cylindrical specimens (ISRM, 1978; ASTM, 2004).

The most popular indirect tensile strength test for testing rocks and concretes is the so-called Brazilian test (Fig. 1a), which was independently proposed by Akazawa in 1943 as a PhD thesis (Machida, 1975; Akazawa, 1943; Fairbairn and Ulm, 2002) and by Carneiro in 1943 at the Fifth Meeting at of the Brazilian Association for Technical Rules Standardization (Carneiro, 1943; Carneiro and Barcellos, 1953; Fairbairn and Ulm, 2002). The testing

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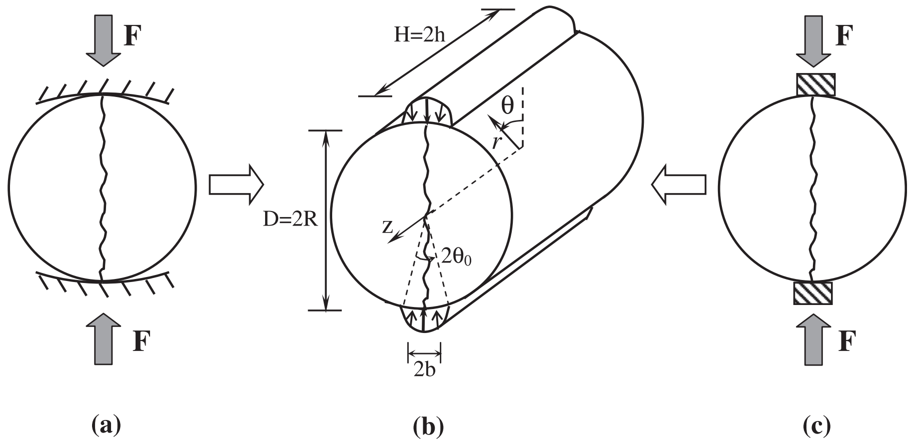

Fig. 1. A finite solid circular cylinder subjected to the indirect tensile test: (a) ISRM Brazilian test; (b) Mathematical model; and (c) ASTM test.

procedure for indirect tensile test was standardized by International Society for Rock Mechanics (Bieniawski and Hawkes, 1978; ISRM, 1978), while the most commonly used indirect tensile test for concrete was standardized by American Society for Testing and Materials (ASTM, 2004). As shown in Fig. 1, finite circular solid cylinders of length *H* and diameter *D* are used for both the ISRM indirect tensile test and the ASTM indirect tensile. Although two diametral strips of loading are adopted for both ISRM and ASTM tests, the loading platens are of different shapes. The recommended height-to-diameter ratios (*H*/*D*) for both ISRM (1978) and ASTM (2004) standards are 0.5. Note that the old standard of ASTM (1995) for bituminous material recommended $H/D = 0.625$ but it was withdrawn in 2003. The failure mode of cylindrical specimens under both ISRM and ASTM tests is always in brittle splitting along the plane formed by joining the two loading strips, as illustrated by the vertical lines shown in Fig. 1. Indirect tensile test is sometimes referred as the "splitting test" (Rocco et al., 1999, 2001). In this paper, the stress distribution for both contact conditions proposed by ISRM and ASTM shown in Fig. 1a and c will be considered.

An analytical solution for a solid circular cylinder subjected to two concentrated diametral line loads was derived by Hertz in 1883 (p. 124, Timoshenko and Goodier, 1982). The main feature of this solution is that a uniform tensile stress is predicted on the vertical plane formed by joining the two line loads. This uniform tensile stress is found equal to $2F_1/(\pi DH)$, where F_1 is the total applied force and *D* and *H* are the diameter and length of the cylinder respectively. Indeed, circular cylinders did fail in tension between these two line loads in all brittle materials (see Fig. 1). Not surprisingly, this simple and elegant solution has been adopted in the standard testing procedures proposed by both ISRM and ASTM. Hondros (1959) extended the solution to the case of applied load being modeled as uniformly distributed strip loads. The 2-D stress components by Hondros (1959) are summarized in Eqs. (5) and (6) of Section 10.7 of Jaeger and Cook (1976) and reproduced here in Appendix B for the sake of completeness. Both of these two-dimensional (2-D) solutions are valid for either very long cylinders (plane strain condition) or very short cylinders (plane stress condition). However, the suggested *H*/*D* value in both ISRM and ASTM standards is 0.5 (ISRM, 1978; ASTM, 2004). It seems that this value may not fully justify the use of the 2-D solution. Indeed, it is more often found that the experimental results cannot be well described by the 2-D analytical solution (Chen and Chen, 1976; Mamlouk et al., 1983; Rocco et al., 2001; Yu et al., 2006).

Therefore, finite element method (FEM) has been employed to study the stress distribution within finite circular cylinders under the indirect tensile test. For example, Yu et al. (2006) studied the shape effect in the Brazilian test using 3D FEM. Numerical results show that for a fixed Poisson's ratio of 0.22 the tensile stress distribution along both the compressed diameter and thickness is not uniform, and the tensile stress near the end surface of the specimen is higher than that of the inner part. It was also found that the 2-D solution by Hertz in 1883 and by Hondros (1959) is not accurate enough to calculate the tensile strength of rocks, especially for relatively thick cylinders. Roque and Buttlar (1992) applied FEM to analyze the indirect tensile test for asphalts and demonstrated that there is a significant variation of the tensile stress along the thickness of the cylinder. Moreover, the two dimensional solutions by Hertz in 1883 and by Hondros (1959) also fail to consider the Poisson effect.

The main objective of this study is to obtain a three-dimensional (3-D) analytical solution for the indirect tensile test, and through this new solution to investigate the validity of 2-D solution in applying to indirect tensile tests. The method of solutions follows the displacement function approach (Muki, 1960; Chau and Wei, 2000, 2001) in converting the coupled equilibrium equations for displacements to a system of two uncoupled differential equations of biharmonic equation and Laplace equation. In cylindrical coordinate, the general solutions of these two displacement functions are expressed in terms of series solution consisting of Bessel functions, hyperbolic functions, and trigonometric functions. In fact, the most difficult step in the solution technique is to assume an appropriate form of solution such that all boundary conditions can be satisfied exactly. In order to satisfy the boundary conditions, Fourier–Bessel expansion technique is applied to expand the applied traction on the curved surface.

The present solution provides a theoretical basis for the stress analysis of and strength interpretation for the commonly adopted indirect tensile strength tests. In view of the popularity of the indirect tensile test in applying to various engineering materials, such as concrete, rocks and asphalts, the present solution is of fundamental importance to the area of material testing. The present 3-D solution also provides a major improvement over the 2-D solution of Hertz in 1883 (Timoshenko and Goodier, 1982) and Hondros (1959), and can be used to examine the effect of Poisson's ratio and shape effect of the specimen on the stress distribution within finite circular cylinders subjected to the indirect tensile test.

2. Mathematical formulation

Fig. 1 shows the typical experimental setup for the ISRM indirect tensile test (Fig. 1a) and the ASTM indirect tensile test

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