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Effective permeability of cracked unsaturated porous materials

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ABSTRACT

This study focuses on a theoretical estimation of the effective permeability of unsaturated cracked porous media. The closed-form flow solution around and in a superconductive crack, embedded in an infinite porous matrix under a far-field condition, is recalled first. Then the solution of flow around a completely unsaturated (empty) crack that is considered as an obstruction against the flow is determined. The flow solution for partially saturated crack in special configurations is obtained by superposition of the two basic solutions for superconductive and empty cracks. The contribution of an unsaturated crack, with a given saturation degree, to the effective permeability is estimated by using dilute upscaling scheme. Numerical results obtained by Finite Elements Method, are in good agreement with the theoretical results for weak crack densities but show the additional effect of cracks interaction for higher densities.

1. Introduction

Evaluation of the effective permeability of cracked porous media is of great interest for geotechnical, geo-environmental and petroleum engineering applications. A literature survey shows that the permeability of unsaturated porous materials has been intensively investigated experimentally and theoretically in recent decades (e.g. van Genuchten, 1980; Fredlund et al., 1994; Zimmerman and Bodvarsson, 1990; Wu and Pan, 2003). The effective permeability of saturated cracked porous materials has also been investigated by various theoretical and numerical approaches (Barenblatt et al., 1960; Bogdanov et al., 2003; Dormieux and Kondo, 2004; Pouya and Ghabezloo, 2010; Pouya and Vu, 2012a,b; Vu et al., 2012). But it seems that far less attention has been paid to modeling the permeability of cracked unsaturated porous (CUP) materials. It should be noted that a fluid-filled crack tends to increase the permeability, whereas an empty crack (crack that does not contain any fluid) acts as an impermeable membrane that constitutes an obstacle to the flow and thus decreases the effective permeability in the direction orthogonal to its plane. To our knowledge, there is not any model representing this double nature contribution of unsaturated cracks to the effective permeability of CUP materials. This paper establishes a solution for flow around an unsaturated crack in an infinite porous matrix that allows a first attempt for estimating this contribution.

The solution of flow in a porous medium containing an elliptical inclusion under a uniform far-field pressure gradient, when the matrix and inclusion obey to the Darcy's law, is well-known and has been applied to different problems (Obdam and Veling, 1987; Zimmerman, 1996; Veling, 2012). However, this solution has not been extended, and it is hard to imagine how it could be extended, to the case of unsaturated cracks with a fluid-filled part and an empty part. Pouya and Ghabezloo (2010) proposed recently a potential solution, expressed as a singular integral equation, for flow around a family of crack-lines in a porous medium with possible intersections in which the fluid flow is modeled by Poiseuille's law. This equation, when written for a straight superconductive crack-line, allows establishing a closed-form solution for flow around this crack. The flow solution around an empty crack (without fluid) in an infinite unsaturated porous matrix can be then deduced from a simple transformation of the previous result. It should be noted that the expression *empty crack* in this paper (different from the *void crack* used in Pouya and Ghabezloo, 2010) is used for a crack without solid fill material, and also without fluid. It acts as an impervious panel against the flow crossing it but has no effect on the flow parallel to it. In this paper we consider only these two cases of superconductive or empty cracks. The intermediate case of a crack with a finite transversal conductivity is not necessary for the purposes of this paper, but has been considered in Vernerey (2011, 2012).

A permeability model of unsaturated cracked material must express the effective permeability as a function of the global saturation degree of the material. However, the global saturation degree reveals to be insufficient to determine the effective permeability. As a matter of fact, different configurations of fluid repartition in

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cracks can correspond to a same global saturation degree but to different effective permeabilities. Specific observation data are yet necessary to make a clearer idea on the fluid repartition model in the cracks of an unsaturated porous material and its relation to the effective permeability. In the absence of direct observation data, physical considerations, mainly based on capillary effects, allow constraining the fluid repartition models. It is well known that these effects imply that in porous materials small pores are filled before greater ones. The same considerations has brought some authors to suppose that in a partially saturated crack, the fluid fills first the cracks extremities where the crack aperture is smaller. When a fluid flow takes place in fractured rock masses, with fracture apertures attaining the range of centimeters to decimeters, it can be conceived that the fluid fill only partially the thickness of the fracture. But in micro-cracks in porous materials, on which this paper focuses, the capillary effects do not allow this type of fluid repartition because it implies a great fluid–gas interface surface. For this reason, the partially saturated crack in this paper is supposed to be divided in fully filled parts in which the whole thickness of the crack is occupied with the fluid, and empty parts occupied by the gas phase (air in most cases).

In this paper, the solution for a partially fluid-filled crack in an infinite porous matrix is deduced from the saturated and empty cracks solutions in some simple and basic configurations of fluid repartition in the crack. Then, based on some simplifying assumptions that are explicitly expressed, this solution is used for a first estimate of the effective permeability of CUP materials by theoretical upscaling methods. The results are discussed and compared to numerical results that allow better taking into account the cracks interactions.

2. Governing equations and basic solutions

2.1. Fluid repartition in the pores and cracks spaces

To determine the effective permeability of a CUP material, a basic information is necessary that concerns the distribution of the fluid content in the matrix pores and cracks spaces. Assuming that the opening of the cracks is bigger than the size of the pores and considering the capillarity phenomena, one can reasonably admit that in a CUP material which is in hydraulic equilibrium, the cracks remain empty until the matrix is fully saturated with fluid. After the matrix is completely saturated, the additional fluid volume contributes to filling the cracks up to a complete saturation of the medium (Fig. 1). This is the same assumption that was made by Wang and Narasimhan (1985) for the conceptual model of flow in unsaturated fractured permeable rock. This model has been widely used for numerical simulations and field applications (Peters and Klavetter, 1988; Niemi and Bodvarsson, 1988; Martinez et al., 1992; Therrien and Sudicky, 1996; Wu et al., 1999; Mathias et al., 2005) to characterize flow and contaminant transport in fractured geological media. Moreover, this model has been also verified by several laboratory or field studies (Gvirtzman et al., 1988; Hodnett and Bell, 1990; Keller, 1996; Wang et al., 1999; Brouyère et al., 2004).

The fluid repartition along the crack-lines is not well known in the phase of saturated matrix and partially saturated cracks. For the same quantity of fluid in the cracks network, different repartition models can be assumed that lead to different effective permeabilities for the CUP material. To deal with this difficulty, which is due to a lack of physical observation data on the fluid repartition model in the cracks, two different cases will be considered and studied in the following sections.

For the hydraulic conductivity of cracks that are supposed without solid fill materials, two basic cases are considered:

– *Superconductive crack:* The crack is fully filled with fluid and is supposed to have an infinite conductivity and no pressure gap between the two faces of the crack.
– *Empty crack:* The crack does not contain any liquid phase fluid, does not contribute to tangent flow. It does not allow transverse flow and acts as an impervious membrane against this flow. A pressure gap can exist between the two faces of the crack.

Then a third case of unsaturated crack is considered in which the crack is constituted of a fluid filled part and an empty part having respectively the properties of superconductive and empty cracks.

2.2. Governing equations

A fundamental problem for modeling flow in cracked porous media is that of the pressure field around a single straight crack in an infinite homogeneous matrix under a far-field condition. The solution of this problem is the key for the resolution of more general problems and in particular for upscaling the effective permeability of cracked porous materials. A potential solution based on singular integral equation is proposed firstly by Liolios and Exadaktylos (2006) to study the steady-state flow around non-intersecting fractures embedded in an infinite porous medium with homogeneous permeability matrix. This solution was extended to the case of the anisotropic matrix containing intersecting fractures by Pouya and Ghabezloo (2010). The transient fluid flow solution around a semi-infinite straight crack in an infinite isotropic matrix was derived recently by Exadaktylos (2012). Here we recall the basic assumptions and results for steady state flow. The list of principal symbols used in the following sections is given in Table 1 at the end of the paper.

Consider an infinite matrix Ω with permeability $\boldsymbol{k}$ containing several intersecting cracks Γ^i prescribed by a pressure field $p_\infty(\underline{x})$ at infinity. Fluid velocity $\underline{v}(\underline{x})$ in the matrix is given by Darcy's law:

$$\forall \underline{x} \in \Omega - \Gamma; \quad \underline{v}(\underline{x}) = -\boldsymbol{k}(\underline{x}) \cdot \underline{\nabla} p(\underline{x}) \tag{1}$$

where p designates the pressure. In the absence of point sources, mass conservation of an incompressible fluid in the matrix reads:

$$\forall \underline{x} \in \Omega - \Gamma; \quad \underline{\nabla} \cdot \underline{v}(\underline{x}) = 0 \tag{2}$$

For the case of conductive cracks, the discharge through the crack is denoted by $q(s)$. The crack-matrix mass exchange law at a point on the crack excluding extremities is obtained by considering mass balance in a portion of the crack comprised between abscissa s and $s + ds$ (Fig. 2):

$$\forall \underline{z}(s) \in \Gamma; \quad [[\underline{v}(\underline{z})]] \cdot \underline{n}(s) + \partial_s q(s) = 0 \tag{3}$$

In this relation, $\underline{z}$ is the point on the crack at the abscissa s, $\underline{n}(s)$ is the normal unit vector to the crack line oriented from Γ^- to Γ^+ (Fig. 2) and $[[\cdot]]$ designates the discontinuity or jump across the crack: $[[\underline{v}(\underline{z})]] = \underline{v}^+(\underline{z}) - \underline{v}^-(\underline{z})$.

The flow in the cracks is described commonly by a Poiseuille type law in which the discharge q is proportional to the pressure gradient along the crack line:

$$\forall s \in \Gamma; \quad q(s) = -c(s)\partial_s p \tag{4}$$

where c is the crack conductivity. The assumption of a laminar flow between two infinite and parallel planes leads to the classical *cubic law* relating this conductivity to the crack hydraulic aperture e and the fluid dynamic viscosity μ by: $c = e^3/(12\mu)$. The limit case of *superconductive fully filled cracks* ($c \to \infty$) will be considered in this paper. In this case, the pressure along the crack line remains constant.