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Evaluation of contact stress during rolling process, by three dimensional analytical inverse method

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ABSTRACT

A fully analytical three-dimensional inverse method has been developed in order to evaluate contact stress in the roll bite during rolling process. Stress measurements can be done by inserting fibre optics inside the roll body, (however no real measurements were available and simulations have been used instead). The inverse method takes as inputs three independent measured (or simulated) components of the stress tensor under the surface of the roll, and evaluates surface tractions of the roll especially in the contact in the roll gap. Stress, deformation and displacements can be obtained in the whole roll as well. This approach uses the theory of 3D isotropic elasticity and relies on displacement harmonic and bi-harmonic potentials expanded into a double Fourier series along the circumferential and axial directions. The identification of the solution involves matrices of size (3×3) computed off-line. This simple solution involves mainly the on-line computation of Fast Fourier Transform (fft) of three inputs, which takes 0.5 s for each (processor 2.8 GHz, time displayed by Scilab 5.3). Good accuracy is obtained, and the number of sensors along the axial direction is studied. Thus, this work encourages the development of an on-line industrial tool.

1. Introduction

Rolling conditions are more and more severe especially in terms of rolling speeds, large reductions and hard steel grades. At the same time more and more severe quality requirements should also be met. That is why many simulations of the rolling process, combining thermo-visco-plastic modeling of the strip and thermo-elastic modeling of the roll, have been proposed. For example Jiang and Tieu (2001) proposed a rigid plastic/visco-plastic FEM. Moreover Montmitonnet (2006) proposed a review of numerical models for hot and cold rolling process. Abdelkhalek et al. (2011) used the older work of Hacquin (1996) and added the computation of the post-bite buckling of the strip, in order to predict accurately flatness defects. Moreover Shahani et al. (2009) simulated a hot rolling process of aluminum by FEM and used an artificial neural network in order to predict the behaviour of the strip during the rolling process (the artificial neural network being trained by the simulation). Considering the highly three-dimensional problem all these works are adapted for a 3D interpretation. Indeed, flatness defects origin is the difference between the incoming strip profile and the work roll deformed profile. Therefore a good control of flatness defects implies a precise evaluation of three-dimensional deformations of the work roll. Elastic deformation is mainly due to the unknown pressure and friction profiles occurring in the roll bite (and depending on lubrication) and the bending moment along the axial direction ensuring equilibrium. Thermal deformations are due to temperature gradients resulting from heat transfers around the roll (conduction in the roll bite, forced convection for the cooling system, and free convection by the surrounding air). In this paper only the elastic deformation is considered. Planar deformations are often assumed near the middle of the roll, and planar stresses at the edges. However, the roll being wider than the strip a significant bending moment along the axial direction makes the elastic problem fully three-dimensional between the middle and the edges of the roll. An accurate elastic profile of the roll is therefore necessarily derived from a 3D solution.

Predictive models are very useful to design flatness control devices. However, controlling and monitoring flatness, thickness or lubrication in real time during the rolling process by a close-loop is aimed. Therefore, measurements interpretation for the evaluation of contact stress in the roll gap is one of the major problem for the rolling industry. Since the contact between the strip and the roll should not be altered, inverse methods are needed.

Among a few two-dimensional contributions one can mention the work of Schnur and Zabaras (1990) who developed an iterative Finite Element Method which minimizes the gap between computed and measured displacements. The work deals with any 2D shapes with sensors under the surface. It is applied to rolling process with very easy conditions, and the reconstruction of surface

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Nomenclature

R_s	outer radius (radius of the surface of the roll)
R_m	inner radius (radius of the measurements)
l	half length of the roll barrel
l	half width of the strip
E	Young modulus of the roll
ν	Poisson modulus of the roll
μ_0	friction coefficient of simulations
$\lambda,\ \mu$	Lamé's coefficients of the roll
$r,\ \theta,\ z$	cylindrical coordinates
$\mathbf{e}_r,\ \mathbf{e}_\theta,\ \mathbf{e}_z$	cylindrical directions
$x,\ y,\ z$	Cartesian coordinates
$\mathbf{e}_x,\ \mathbf{e}_y,\ \mathbf{e}_z$	Cartesian directions
ω	rotation speed of the roll
f	frequency of acquisition of measured stresses
$(..)$	$rr, r\theta, rz, \theta\theta, \theta z$ or zz
u_r	Radial displacement
u_θ	circumferential displacement
u_z	axial displacement
I_n	modified Bessel function of n-order
Φ	bi-harmonic displacement potential
$\Psi, \phi, \gamma, \psi, \phi^*, \gamma^*, \psi^*$	harmonic displacement potentials
$\sigma^T_{(..)}$	total stress tensor
$\sigma_{(..)}$	stress tensor due to Φ, Ψ, ϕ, γ and ψ
$\sigma^*_{(..)}$	stress tensor due to ϕ^*, γ^* and ψ^*
$\sigma^m_{(..)}$	measured stresses
$\sigma^{(..)}_{n,p}$	Fourier coefficient of $\sigma_{(..)}$
$S^{(..)}_{n,p}$	Fourier coefficient of $\sigma^m_{(..)}$
$\Phi^n_p, \widetilde{\Phi}^n_p$	unknown coefficients in the expansion of Φ
Ψ^n_p	unknown coefficient in the expansion of Ψ
$A_n,\ B_n,\ C_n$	unknown coefficients in the expansion of $\phi,\ \gamma$ and ψ
α_k	unknown coefficients in the expansion of $\phi^*,\ \gamma^*$ and ψ^* ($k \in \{1;2;3;4\}$)
N	truncation number according to n
P	truncation number according to p
$\boldsymbol{M}^{n,p}(r)$	matrix of size (3×3)
$\kappa_{n,p}$	condition number of $\mathbf{M}^{n,p}(r)$
$\delta_{n,p}$	mean error in the calculation of $S^{(..)}_{n,p}$
T^+	resultant traction at $z = L$
$V^+_x,\ V^+_y$	resultant shears according to x or y coordinates at $z = L$
$M^+_x,\ M^+_y,\ M^+_z$	resultant bending moments according to $x,\ y$ or z coordinates at $z = L$
$\epsilon_{(..)}$	reconstruction error (percentage)
N_z	number of sensors considered along the axial direction

tractions is not very accurate. Bezerra and Saigal (1995) developed also an iterative Boundary Element Method (BEM), which interprets the same rolling conditions. However, computation times are long. On the other hand, Meierhofer and Stelson (1987) proposed two-dimensional analytical developments. The method involves an expansion of the Airy potential into a power series (with additional terms) and an expansion of the stress measured inside the roll into a Fourier series. Then, an identification of the coefficients of both series is needed. To obtain enough equations, the authors used measurements on two concentric circles. They proposed measuring the stresses at two different points and at two different radii (which is technologically very difficult for industrial mills). Moreover, many inversions of matrices of identification (well-conditioned if the two radii are sufficiently distinct from each other) are needed. More recently Legrand et al. (2012b) computed this method and investigated its effectiveness by simulating several industrial conditions. The authors concluded that, for common industrial rolling conditions, the method developed by Meierhofer and Stelson (1987) is quite accurate but hardly usable because of the very long computation time. Moreover, the authors demonstrate that the contact stress cannot be reconstructed precisely under temper-rolling conditions (with extremely sharp stress gradients and a very short roll bite). The recent contribution of Weisz-Patrault et al. (2011) is successful under this kind of condition and needs only one measurement point inside the roll body. Moreover, the computation times are very short (0.07 s/cycle). Weisz-Patrault et al. (2012a) proposed also a fast analytical thermal inverse method (0.05 s/cycle), tested experimentally (Weisz-Patrault et al., 2012b; Legrand et al., 2012a, 2013). Both elastic and thermal solutions are coupled analytically by Weisz-Patrault et al. (2013) in order to compute thermal stress (fatigue) and thermal expansion.

All these works are two-dimensional. There are only very few attempts (or maybe none) to develop three-dimensional inverse methods adapted for rolling process. This is mainly due to the fact that numerical iterative formulations involve extremely long computation times and stack size memory problems as mentioned by Loulou and Artioukhine (2006). For analytical solutions the challenge is that complex analysis cannot be used in 3D, and quaternionic formulations are developed but not used practically (Kutrunov, 1992; Tsalik, 1995; Bock and Gürlebeck, 2009). However different methods have been developed for other kinds of elastic 3D inverse problems. For example Andrieux and Baranger (2008) proposed an energy error-based method for 3D inverse Cauchy problems (both displacement and surface tractions are known for a part of the boundary and the remaining part should be determined). Delvare et al. (2010) proposed an iterative 3D inverse method also for Cauchy problems. A 3D inverse Finite Element Method adapted for frame structures has been developed by Gherlone et al. (2012).

However, inverse methods for rolling process do not belong exactly to these class of problems. In the field of industrial inverse method adapted for rolling process, sensors sufficiently robust to resist high pressure and temperature levels are in development. Thus, stress measurements by fibre optics are studied (for instance by Ferdinand et al., 2009), and the first attempts are naturally two-dimensional. However, industrial work rolls can be equipped with several stress sensors under the surface, and three-dimensional inverse method should be developed for measurement interpretation.

2. Choice of the method

The main idea of this contribution is to solve analytically the direct problem of a finite circular cylinder loaded by unknown surface tractions for which an analytical form (expansion into series) is given. The coefficients are determined with stress measurements inside the roll body. Equivalently, the direct problem can be solved analytically for a sub-domain (in this paper: a circular cylinder of radius R_m) then the solution (valid in the whole roll) is evaluated at the surface. Therefore, an efficient direct analytical method is needed.

A very general solution for the Neuman problem (surface tractions imposed at the surface) for a general elastic body is proposed in the book of Bui (2006). The solution relies on the Kelvin–Somigliana tensor (equivalent to the Green tensor for elastostatic)