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Issues of dynamics of conductive plate in a longitudinal magnetic field

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ABSTRACT

On the basis of Kirchhoff hypothesis the problem of vibrations of conductive plate in a longitudinal magnetic field is brought to the solution of the singular integral–differential equation with ordinary boundary conditions. The formulated boundary-value problem solved and the influence of magnetic field on the characteristics of vibration process of the examined magnetoelastic system is investigated. Via the analysis of obtained solutions it is shown that the presence of magnetic field can: (a) increase essentially the frequency of free magnetoelastic vibrations of the plate; (b) decrease essentially the amplitude of forced vibrations if $r \leqslant 1$, where $r = \theta/\omega, \theta$ – is the frequency of acting force, ω – is the frequency of own vibrations of the plate magnetic field is being absent; (c) increase essentially the amplitude of forced vibrations if $r > 1$; (d) decrease essentially the width of main areas of dynamic instability. It is shown that: (1) in the case of perfectly conductive plates magnetic field constricts essentially the width of main area of dynamic instability; (2) if plate's material has the finite electroconductivity, then the certain value of the intensity of external magnetic field exists, exceeding of which excludes the possibility of appearance of parametric type resonance. It is shown also that in dependence on the character of initial excitements the plate can vibrate either across the initial non-deformable state, or across the initial bent state.

1. Introduction

Review on the papers and books from the presented field of interest one can find in the books Baghdasaryan (1999), Ambartsumian and Baghdasaryan (1996), Maugin (1988), Podstrigach et al. (1982). In these books one can find the main citations to the reference of the theory of magnetoelasticity. Investigations in this field have widely evolved in recent years also. These investigations are mainly devoted to the problems of free vibrations, for example, Baghdasaryan et al. (2003) and Librescu et al. (2004). The Cauchy problems are not addressed yet and investigations devoted to the forced and parametric type vibrations are almost absent. The presented work is devoted to the investigation of dynamic processes in electrically conductive plated interacting with magnetic fields. This paper is consists of two parts: the dynamic processes in the case of perfectly conductive plate are investigated in the first part; and the finiteness of the coefficient of electroconductivity of plate's material (finite electroconductive plate) is taken into account in the second part of the paper.

As it was noted above, the presented paper is devoted to the investigation of dynamic processes in conductive plates in presence of magnetic field. On the basis of such investigations the following assumptions are taken into account: (a) the models of perfectly conductors; (b) Kirchhoff hypothesis. On the basis of these assumptions the governing two-dimensional equations and corresponding conditions were obtained by several authors, such as Kaliski (1962) and Baghdasaryan and Belubekyan (1967)). These equations characterize vibrations of perfectly conductive plates in external magnetic field. The limits of tangential components of induced in plate's surroundings are included in these equations. Thus, the addressed boundary-value problems must be studies together with Maxwell equations, in presence of coupling conditions at the surface of the plate. Due to these insurmountable mathematical difficulties these problems were solved either in the case of infinitely long plate, or the approximate solutions were used. In particular, the asymptotical method to solve the boundary-value problem was used (Baghdasaryan, 1986). In the case, when magnetic field is absent, the above-mentioned method is proposed in Bolotin et al. (1960). Using the basic postulates of the potential theory, in the case of two-dimensional problem the limits of tangential components of the induced in plate magnetic field are obtained in the presented paper. As a result the investigation of the noted vibration process is brought to the solution of initial-boundary-value problem for integral–differential equation with singular kernel. Thus it was able to avoid from approximate solution of external problem and it was established an opportunity to solve not only the problems of natural magnetoelastic vibrations, but the problems of forced and parametric vibrations in presence

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of external magnetic fields, too. It was possible to solve the problems with initial conditions also.

2. Basic equations

Let an elastic isotropic perfectly conductive plate of constant thickness $2h$ referred to the Cartesian system of coordinates x_1, x_2, x_3 so that the middle plane of non-deformed plate coincides with the coordinate plane x_1, x_2. The plate is compressed via the force with the intensity $P(t) = P_0 + P_1 \cos\theta t$ and occupies the area $(|x_1| \leqslant a, |x_2| < \infty, |x_3| < h)$ and under the transversal load $Q(x_1, t) = Q(-x_1, t)$ oscillates in an external constant magnetic field with a given intensity vector $\vec{H}(H_{01}, 0, 0)$. Boundary conditions at the edges of the plate $x_1 = \pm a$ addressed such that the plate oscillates in the form of a cylindrical surface with generators parallel to the coordinate line $0x_2$ (all quantities are independent of the coordinate x_2). It is assumed that the plate does symmetrical oscillations, i.e. the deflection of the plate $w(x_1, t)$ – is an even function of x_1 in the area $[-a, a]$.

To investigate the oscillations of the examined magnetoelastic system the basic assumptions of the theory of linear magnetoelasticity will be used, assuming the hypothesis of undeformable normal is true. According to this hypothesis the following known relations for the components of elastic displacements $\vec{u}(u_1, 0, u_3)$ can be presented:

$$u_1 = u - x_3 \frac{\partial w}{\partial x_3}, \quad u_2 = 0, \quad u_3 = w \tag{1.1}$$

where $u(x_1, t), w(x_1, t)$ – are unknown displacements of the points of middle plane of the plate. It is also assumed that magnetic permeability of plate's material is equal to zero and the influence of tangential forces of inertia and displacement currents on the characteristics of plate's vibrations can be neglected.

According to the above said and using the results of Refs. Baghdasaryan (1983), the equation of transverse magnetoelastic vibrations of the examined plate has the form:

$$\begin{aligned} &D\frac{\partial^4 w}{\partial x_1^4} + 2\rho h \frac{\partial^2 w}{\partial t^2} + 2\rho h\varepsilon \frac{\partial w}{\partial t} + P(t)\frac{\partial^2 w}{\partial x_1^2} \\ &+ \frac{H_0}{4\pi}\int_{-h}^{h}\left(\frac{\partial h_1}{\partial x_3} - \frac{\partial h_3}{\partial x_1}\right)dx_3 + \frac{H_0}{4\pi}\left[h_1^{(e)+} - h_1^{(e)-}\right] = Q(x_1, t). \end{aligned} \tag{1.2}$$

where $D = 2Eh^3/3(1-\nu^2)$ – is the flexural rigidity, E – is elasticity modulus of plate material, ν – is the Poisson's ratio ρ – is the density, ε – is the coefficient of linear damping of the material; $h_1^{(e)\pm}$ – are unknown boundary values of the tangential component of $h_1^{(e)}(x_1, x_3, t)$ induced in the external medium (in vacuum) magnetic field $\vec{h}_1^{(e)}(h_1^{(e)}, 0, h_3^{(e)})$ on surfaces $x_3 = \pm h$ of the plate, respectively; $h_1(x_1, x_3, t)$ and $h_3(x_1, x_3, t)$ – are components of induced in the plate magnetic field $\vec{h}(h_1, 0, h_3)$.

According to the accepted assumptions the determination of included in (1.2) magnetic quantities $h_1^{(e)\pm}, h_1$ and h_2 is brought to the joint solution of equations of quasi-static electrodynamics in an external medium

$$\operatorname{rot}\vec{h}^{(e)} = 0, \quad \operatorname{div}\vec{h}^{(e)} = 0 \tag{1.3}$$

with the equations of electrodynamics of perfectly conductive bodies in the plate's area

$$\frac{\partial \vec{h}}{\partial t} = \operatorname{rot}\left(\frac{\partial \vec{u}}{\partial t} \times \vec{H}\right) \tag{1.4}$$

under the conditions

$$h_3^{(e)} = h_3 \quad \text{for} \quad x_3 = \pm h, \ |x_1| \leqslant a \tag{1.5}$$

$$h_1^{(e)} = h_1 \quad \text{for} \quad x_1 = \pm a, \ |x_3| \leqslant h \tag{1.6}$$

on the surfaces of the plate under the conditions

$$h_i = 0 \quad \text{for } t = t_0 (i = 1, 3) \tag{1.7}$$

and the conditions of attenuation of excitements on infinity

$$h_i = 0 \quad \text{for} \quad x_1^2 + x_2^2 + x_3^2 \to \infty \tag{1.8}$$

From (1.4) and (1.7), in account of (1.1) one can find

$$h_1 = 0, h_3 = H_0 \frac{\partial W}{\partial x_1} \quad \text{for} \quad |x_1| \leqslant a, \ |x_3| \leqslant h \tag{1.9}$$

where $W = w(x_1, t) - w(x_1, t_0)$.

As the plate is thin-walled, it is assumed, that the plate can be presented via the surface, in which the component $h_1^{(e)}$ of the vector $\vec{h}_1^{(e)}$ has discontinuity. Besides this according to the Eqs. (1.3), (1.4), (1.5), (1.6), and (1.9) let us assume, that $h_3^{(e)}$ is an even function on the coordinate x_3 (then $h_1^{(e)}$, according to the condition $div\vec{h}^{(e)} = 0$, will be an odd one with respect to this coordinate). Based on these assumptions the determination of unknowns $h_1^{(e)}$, according to the conditions (1.3), (1.6), and (1.9) can be brought to the following Dirichlet problem for the half-space $x_3 > 0$:

$$\begin{aligned} &\frac{\partial^2 h_1^{(e)}}{\partial x_1^2} + \frac{\partial^2 h_1^{(e)}}{\partial x_3^2} = 0, \\ &h_1^{(e)}\big|_{x_3=0^-} = \begin{cases} h_1^{(e)+}, & |x_1| < a \\ 0, & |x_1| \geqslant a \end{cases} \end{aligned} \tag{1.10}$$

The solution of the problem (1.10) has the form (Sobolev, 1966):

$$h_1^{(e)} = \frac{x_3}{\pi}\int_{-a}^{a}\frac{h_1^{(e)+}(\xi, t)d\xi}{(x_1 - \xi)^2 + x_3^2} \tag{1.11}$$

Due to (1.8) and (1.11) from (1.3) the following expression will be obtained to determine $h_3^{(e)}$:

$$h_3^{(e)} = \frac{1}{\pi}\int_{-a}^{a}\frac{(x_1 - \xi)h_1^{(e)+}(\xi, t)\,d\xi}{(x_1 - \xi)^2 + x_3^2} \tag{1.12}$$

Substituting (1.9) and (1.12) into the surface condition (1.5), for determination $h_1^{(e)+}$ the following singular integral equation with Cauchy type kernel is obtained:

$$\frac{1}{\pi}\int_{-a}^{a}\frac{h_1^{(e)+}(\xi, t)d\xi}{x_1 - \xi} = H_0\frac{\partial W}{\partial x_1} \tag{1.13}$$

Solution of the integral equation (1.13) which satisfies the condition (1.6) and as the function $w(x_1, t)$ is an even one with respect to the coordinate x_1 has the form (Gakhov, 1963)

$$h_1^{(e)+} = \frac{H_0}{\pi}\int_{-a}^{a}\sqrt{\frac{a^2 - x_1^2}{a^2 - \xi^2}}\frac{\partial W}{\partial \xi}\frac{d\xi}{x_1 - \xi} \tag{1.14}$$

The $h_1^{(e)-}$ is determined analogously and one can obtain:

$$h_1^{(e)-} = h_1^{(e)+} \tag{1.15}$$

Substituting (1.9), (1.14), and (1.15) into the (1.2), the investigation of magnetoelastic vibrations of the examined plate to the solution of the following singular integral equation is brought:

$$\begin{aligned} &D\frac{\partial^4 w}{\partial x_1^4} + 2\rho h\frac{\partial^2 w}{\partial t^2} + 2\rho h\varepsilon\frac{\partial w}{\partial t} - \frac{hH_0^2}{2\pi}\frac{\partial^2 W}{\partial x_1^2} \\ &+ \frac{H_0}{2\pi^2}\int_{-a}^{a}\sqrt{\frac{a^2 - x_1^2}{a^2 - \xi^2}}\frac{\partial W}{\partial \xi}\frac{d\xi}{x_1 - \xi} + P(t)\frac{\partial^2 w}{\partial x_1^2} = Q(x_1, t). \end{aligned} \tag{1.16}$$