International Journal of Solids and Structures 50 (2013) 176–185

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Unified series solution for the anti-plane effective magnetoelectroelastic moduli of three-phase fiber composites

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ARTICLE INFO

Article history:
Received 3 February 2012
Received in revised form 25 August 2012
Available online 27 September 2012

Keywords:
Effective properties
Magnetoelectroelastic coupling
Anti-plane
Interphase
Fiber-reinforced composites

ABSTRACT

The anti-plane magnetoelectroelastic behavior of three-phase magnetoelectroelastic composites (fiber/interphase/matrix) with doubly periodic microstructures is dealt with. With the aid of the matrix notation, the anti-plane magnetoelectroelastic coupling problem is formulated as same as the anti-plane piezoelectric coupling problem. And then the eigenfunction expansion-variational method (EEVM) is extended to solve such a problem. Series solutions for the effective magnetoelectroelastic moduli are presented, which are in a unified form for generally periodic fiber arrays, different unit cell shapes as well as different constituent properties, and are applicable for high volume fraction of fibers. With the present solution, it is found that the effective magnetoelectric coefficient of a two-phase composite may have two local extrema rather than only one extremum predicted by the Mori-Tanaka method. By optimizing the volume fraction, permutation and the choice of the constituent phases, the maximum magnitude of the effective magnetoelectric coefficient of a three-phase composite can be much larger than that of any of the two-phase composites, and the sign of the magnetoelectric coefficient can be changed, which is not observed in a two-phase composite. For composites with a generally periodic array of fibers, the effective magnetoelectric moduli can be anisotropic.

1. Introduction

Magnetoelectric effect provides a useful tool for the conversion of energy between magnetic and electric forms. As a successful case of the man-made materials, magnetoelectroelastic composites can exhibit a magnetoelectric effect that is absent in each of the phases, by combining the piezoelectric effect in a piezoelectric phase and the piezomagnetic effect in a piezomagnetic phase. This kind of magnetoelectric effect in magnetoelectroelastic composites is caused by "product properties" (Van Suchtelen, 1972): the electric field and the magnetic field are related through the elastic strain. In a two-phase magnetoelectroelastic composite $BaTiO_3$/$CoFe_2O_4$, the magnetoelectric coefficient can be two orders of magnitude larger than that of the single-phase magnetoelectric materials (Van Run et al., 1974). Moreover, such a magnetoelectric effect in the composite can be observed at room temperature, whereas the magnetoelectric effect in single-phase magnetoelectric materials is often observed only at very low temperature (Spaldin and Fiebig, 2005). Due to such outstanding performances, magnetoelectroelastic composites are increasingly applied in intelligent structures and smart devices (Ma et al., 2011; Nan et al., 2008; Pyu et al., 2002; Ramesh and Spaldin, 2007; Srinivasan, 2010). Motivated by such a finding, various designs of novel magnetoelectroelastic composites are presented, as well as the corresponding theoretical models and fabrication methods are developed (Eerenstein et al., 2006; Nan et al., 2008; Ramesh and Spaldin, 2007; Spaldin and Fiebig, 2005; Srinivasan, 2010).

According to the microstructural connectivity, the magnetoelectroelastic composites are generally categorized into particle composites, fiber composites, laminate composites, and so on (Nan et al., 2008). The magnetoelectroelastic fiber composites have been attracting extensive attention because of their enhanced magnetoelectric performance as well as the still open question in modeling. To achieve the 'tailored' properties (Zohdi, 2008), reasonable models for simulation of the macroscopic and microscopic response are necessary. Some classic micromechanical models for purely elastic problems are generalized to solve the magnetoelectroelastic problems in terms of an analogy between the governing equations of the magnetoelectroelastic problems and the purely elastic problems. These models include the dilute model (Zhang and Soh, 2005), self-consistent model (Nan, 1994; Srinivas and Li, 2005; Zhang and Soh, 2005), generalized self-consistent model

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http://dx.doi.org/10.1016/j.ijsolstr.2012.09.020

(Tong et al., 2008), Mori-Tanaka model (Dinzart and Sabar, 2011; Huang and Kuo, 1997; Li and Dunn, 1998; Srinivas et al., 2006; Wang and Pan, 2007; Wu and Huang, 2000; Zhang and Soh, 2005), composite cylinder assemblage model (Benveniste, 1995) and multi-inclusion model (Li, 2000). With these models some general analytical solutions for effective magnetoelectroelastic properties are presented by treating the inclusion interactions either approximately or in a statistically sense. As coupling moduli resulted from the interaction between the piezoelectric phase and the piezomagnetic phase, the magnetoelectric coefficients strongly depend on the inclusion interactions. From the existing researches, the magnetoelectric coefficient reaches the extremum usually at a relatively high inclusion volume fraction, where the inclusion interaction is strong. In consideration of these, high-order solutions of the magnetoelectric coefficient treating the inclusion interactions more accurately are necessary.

By adding another active interphase between the fiber and matrix of a two-phase composite, the three-phase magnetoelectroelastic fiber composites possess greater design flexibility. Several researches focus on such three-phase composites, such as multi-coated circular fibrous composites (Kuo and Pan, 2011), multi-coated elliptic fibrous composites (Kuo, 2011), composites with thinly coated inclusions (Dinzart and Sabar, 2011). Among these works, Kuo and Pan (2011) found that the magnetoelectric effect in coated composites can be enhanced by more than one order of magnitude as compared to the corresponding two-phase composite. However, an attendant problem of greater design flexibility is that more microscopic parameters influencing the effective properties need to be considered. Therefore, in order to optimize the numerous microscopic parameters to obtain extrema or desired magnitudes of effective magnetoelectroelastic properties, the solutions of the effective properties should cover all the key microscopic parameters.

In contrast to random microstructures, periodic microstructures usually exist in elaborately designed composites, since the design of an advanced composite is generally the one for a unit cell (Sun et al., 2001). Along with the progress of composites fabrication technology, some advanced magnetoelectroelastic composites with relatively rigorous periodic microstructures are invented. Recently, Zheng et al. (2004) reported a self-assembled multiferroic nanocomposite with hexagonal arrays of $CoFe_2O_4$ nanopillars embedded in a $BaTiO_3$ matrix. Boyd IV et al. (2003) presented a method for using arrays of micro-electro-mechanical systems electrodes and electromagnets to achieve microscale positioning of piezoelectric and piezomagnetic particles in liquid polymers. Shi et al. (2005) reported a kind of 1–3-type multiferroic and multi-functional composite with Pb(Zr, Ti)O_3 rod arrays embedded in a ferromagnetic medium of (Tb, Dy)Fe_2/epoxy produced by the dice-and-fill method. On the other hand, the periodic composite models provide useful limiting values of interacting inclusions from entirely disorder (random) to order (Nemat-Nasser and Hori, 1999). As far as the magnetoelectroelastic composites with a periodic array of fibers are concerned, Lee et al. (2005) performed a finite element analysis of a representative volume element to determine the effective magnetoelectroelastic moduli. Kuo and Pan (2011) generalized Rayleigh's formulism for the evaluation of the effective material properties in multicoated circular fibrous multiferroic composites. Camacho-Montes et al. (2009) and Espinosa-Almeyda et al. (2011) applied the asymptotic homogenization method to calculate the properties of the fiber composites and the ones with imperfect interfaces. Kuo (2011) combined the methods of complex potentials with a re-expansion formulae and the generalized Rayleigh's formulation to obtain a complete solution of the multi-field many-inclusion problem. The periodic microstructures considered in these researches are either hexagonal or square fiber arrays. Researches on the magnetoelectroelastic composites with a generally periodic array of fibers are not presented yet. Though, to the best of our knowledge, a real composite with such a periodic microstructure is not reported yet, such composites can be fabricated by the technique presented by Boyd IV et al. (2003). Furthermore, the magnetoelectroelastic composites with a generally periodic array of fibers are expected to exhibit a special magnetoelectric effect due to the overall anisotropy induced by general fiber arrays. That is, an electric field in one direction can result from a magnetic field in another perpendicular direction. Therefore, it is highly desirable to develop a method to analyze the magnetoelectric effect of such composites, especially the influence of the different fiber distributions and the anisotropy induced by general fiber arrays.

The present work is devoted to extend the eigenfunction expansion-variational method (EEVM) (Yan et al., 2011) to solve the anti-plane magnetoelectroelastic coupling problem for composites with a generally doubly periodic array of fibers. Series solutions in unified form for the effective magnetoelectroelastic moduli are presented, and then the validity and efficiency of such series solutions are verified. With the present solution, the influences of the volume fraction, permutation and the choice of the constituent phases, as well as the fiber distribution on the effective magnetoelectroelastic moduli are discussed. And then the influences of the volume fraction and interphase on interfacial stresses are discussed. Finally, the anisotropy of the composites induced by the general fiber arrays is discussed.

2. Statement and formulation of the problem

Consider a three-phase fiber composite subjected to combined anti-plane shear, inplane (Ox_1x_2-plane) electrical and magnetic loads as shown in Fig. 1, where the fiber, coating (interphase) and matrix are made of piezoelectric materials, piezomagnetic materials or inactive materials. The fibers are aligned in x_3 direction, the piezoelectric and piezomagnetic materials are polarized and magnetized along x_3-axis, respectively. Then only the anti-plane displacement w, inplane electrical potential ϕ and magnetic potential φ need to be considered, they are the functions of x_1 and x_2 only,

$$\{w, \phi, \varphi\} = \{w(x_1, x_2), \phi(x_1, x_2), \varphi(x_1, x_2)\} \tag{1}$$

For transversely isotropic piezoelectric materials and piezomagnetic materials, the anti-plane constitutive equations are

$$\begin{bmatrix} \tau_{i3} \\ D_i \end{bmatrix} = \begin{bmatrix} C_{44} & e_{15} \\ e_{15} & -\kappa_{11} \end{bmatrix} \begin{bmatrix} 2\varepsilon_{i3} \\ -E_i \end{bmatrix}, \begin{bmatrix} \tau_{i3} \\ B_i \end{bmatrix} = \begin{bmatrix} C_{44} & q_{15} \\ q_{15} & -\mu_{11} \end{bmatrix} \begin{bmatrix} 2\varepsilon_{i3} \\ -H_i \end{bmatrix}, \tag{2}$$

respectively. τ_{i3}, D_i and B_i ($i = 1,2$) are the anti-plane shear stress and inplane electrical displacement and magnetic induction components, respectively; ε_{i3}, E_i and H_i ($i = 1,2$) are the strain, electrical field and magnetic field components, respectively; C_{44}, e_{15}, q_{15}, κ_{11} and μ_{11} are the shear modulus, piezoelectric coefficient, piezomagnetic coefficient, dielectric permittivity and magnetic permittivity, respectively. The piezoelectric constitutive equation and piezomagnetic constitutive equation can be cast into the following unified form:

$$\begin{bmatrix} \tau_{i3} \\ D_i \\ B_i \end{bmatrix} = \begin{bmatrix} C_{44} & e_{15} & q_{15} \\ e_{15} & -\kappa_{11} & -a_{11} \\ q_{15} & -a_{11} & -\mu_{11} \end{bmatrix} \begin{bmatrix} 2\varepsilon_{i3} \\ -E_i \\ -H_i \end{bmatrix} \tag{3}$$

where a_{11} is the magnetoelectric coefficient, and is generally zero for monolithic piezoelectric materials and piezomagnetic materials.

For brevity and convenience, introduce the following matrix notations: