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A unified solution for self-equilibrium and super-stability of rhombic truncated regular polyhedral tensegrities

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ABSTRACT

As a novel class of lightweight and reticulated structures, tensegrities have found a diversity of technologically significant applications. In this paper, we theoretically investigate the self-equilibrium and super-stability of rhombic truncated regular polyhedral (TRP) tensegrities. First, the analytical solutions are derived individually for rhombic truncated tetrahedral, cubic, octahedral, dodecahedral, and icosahedral tensegrities. Based on these solutions, we establish a unified analytical expression for rhombic TRP tensegrities. Then the necessary and sufficient condition that ensures the existence of a self-equilibrated and super-stable state is provided. The obtained solutions are helpful not only for the design of self-equilibrated and super-stable tensegrities but also for their applications in biomechanics, civil and aerospace engineering.

1. Introduction

Tensegrity, a structure based on the complementary equilibrium of axial tension and compression, is perceived as a potential solution to many practical problems (Skelton and de Oliveira, 2009). In nature, tensegrity can be considered as a generic principle in organisms ranging from molecules (Luo et al., 2008; Morrison et al., 2011), cells (Holst et al., 2010; Stamenovic and Ingber, 2009) to tissues (Maina, 2007). In industry, tensegrity has a variety of important applications in, for instance, the development of advanced materials (Fraternali et al., 2012), novel civil architectures (Rhode-Barbarigos et al., 2010; Yuan et al., 2007), smart structures and systems (Ali and Smith, 2010; Moored et al., 2011), and deployable devices for aerospace technology (Sultan, 2009).

In the design of a tensegrity structure, two key steps, among others, are self-equilibrium and stability analyses to determine the conditions under which the structure will be self-equilibrated and stable, respectively (Zhang et al., 2012). The existing self-equilibrium analysis methods can be generally classified into two categories, analytical and numerical. Analytical approaches can be used only for simple tensegrities with high symmetry (e.g. Murakami and Nishimura, 2001; Zhang and Ohsaki, 2012), while numerical methods are often invoked for relatively complicated tensegrities (e.g. Estrada et al., 2006; Li et al., 2010b; Pagitz and Tur, 2009). The criterion of super-stability provides a sufficient condition for the stability of a tensegrity structure consisting of conventional material elements with always positive axial stiffness (Guest, 2011; Schenk et al., 2007; Zhang and Ohsaki, 2007). In the present paper, only the static stability is considered, excluding the instability problems caused by non-conservative disturbances. A tensegrity structure is said to be super-stable if it is stable for any level of force densities satisfying the self-equilibrium conditions without causing element material failure (Connelly, 1999; Juan and Tur, 2008). For the self-equilibrated and super-stable tensegrities, increasing the level of force densities normally tend to stiffen and stabilize them (Connelly and Back, 1998). This property is important for the constructions and applications of tensegrities.

In practice, a tensegrity structure is generally modelled as a set of weightless axial compressive elements (called 'bars' or 'struts') and tensile elements ('strings' or 'cables') connected by frictionless spherical joints (Juan and Tur, 2008). One can construct tensegrities by assembling a certain number of elementary cells according to certain design rules (Feng et al., 2010; Li et al., 2010a). Based on the local configuration of each constituent elementary cell, Pugh (1976) defined two major classes of tensegrities, called Z-based (or zig-zag) structures and rhombic (or diamond) structures, respectively (Feng et al., 2010). In the past decade, the self-equilibrium and stability of some Z-based truncated regular polyhedral (TRP) tensegrities have been investigated by using either analytical or numerical methods (e.g. Koohestani, 2012; Li et al., 2010b; Murakami and Nishimura, 2001, 2003; Pandia Raj and Guest, 2006; Zhang and Ohsaki, 2012). Recently Zhang et al. (2012) derived a unified analytical solution for the self-equilibrium and super-stability of all Z-based TRP tensegrities. In recognition to

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their important applications in, for instance, cytoskeleton (Ingber, 2010; Pirentis and Lazopoulos, 2010), the self-equilibrium and stability of rhombic expandable octahedron tensegrities have been studied by some researchers (e.g. Lazopoulos, 2005; Xu and Luo, 2011; Zhang and Ohsaki, 2006). However, the properties of self-equilibrium and super-stability of rhombic TPR tensegrities, an important class of tensegrity structures of extensive interest, are still unclear.

Therefore, the present study aims at exploring the self-equilibrium and stability properties of rhombic TRP tensegrities. The paper is organized as follows. In Section 2, the concepts of Z-based TRP tensegrities and rhombic TRP tensegrities are briefly reviewed. Section 3 gives the theoretical basis for the self-equilibrium and super-stability of tensegrities. Sections 4 and 5 analyze, respectively, the self-equilibrium and super-stability of rhombic truncated tetrahedral, cubic/octahedral, and dodecahedral/icosahedral tensegrities. Section 6 establishes a unified solution for the necessary and sufficient condition that ensures the existence of self-equilibrated and super-stable states for all types of rhombic TRP tensegrities.

2. TRP tensegrities

2.1. Z-based TRP tensegrities

To facilitate subsequent analysis, we refer to the following definition of polyhedra (Coxeter, 1973):

Definition 1. A regular polyhedron can be uniquely identified by the Schläfli symbol $\{n, m\}$, where n is the number of edges in each face and m is the number of faces around each vertex.

In total, there are five types of convex regular polyhedra (Cromwell, 1997), including tetrahedron identified by $\{3, 3\}$, cube $\{4, 3\}$, octahedron $\{3, 4\}$, dodecahedron $\{5, 3\}$, and icosahedron $\{3, 5\}$, as shown in Fig. 1(a). By cutting off each vertex of these regular polyhedra, one can obtain the five types of truncated regular polyhedra, as shown in Fig. 1(b).

In a Z-based TRP tensegrity structure, each string corresponds to an edge of the truncated regular polyhedron and the bars connect the vertexes by following the rule of Z-shaped elementary cells (Li et al., 2010a). For illustration, we take the construction of a Z-based truncated tetrahedral tensegrity as an example. In the first step, one truncates a tetrahedron by cutting all its original vertices and creating a new polygonal facet around each vertex. Fig. 2(a) shows the vertices and edges of the truncated tetrahedron. Then, the strings and bars are added following the procedure proposed by Li et al. (2010a). Fig. 2(b) illustrates the nodes, strings, and bars of the tensegrity, where a Z-shaped cell, consisting of the nodes 1–4, is highlighted. Fig. 1(c) gives the five types of Z-based TRP tensegrities corresponding to the polyhedra in Fig. 1(a) and the truncated polyhedra in Fig. 1(b).

2.2. Rhombic TRP tensegrities

A rhombic cell in self-equilibrated tensegrities has the similar load-bearing feature as a Z-shaped cell (Feng et al., 2010), as shown in Fig. 3. In both elements, the external forces should be applied in a certain range of direction such that the bar is under compression and the strings are under tension. In other words, the nodes 1 and 3 tend to approach each other while the nodes 2 and 4 tend to separate. Therefore, a rhombic tensegrity structure can be simply constructed from a Z-based tensegrity structure by simply replacing all its Z-shaped cells with rhombic cells. For example, based on the Z-based truncated tetrahedral tensegrity in Fig. 2(b), a rhombic truncated tetrahedral tensegrity can be readily built, as shown in

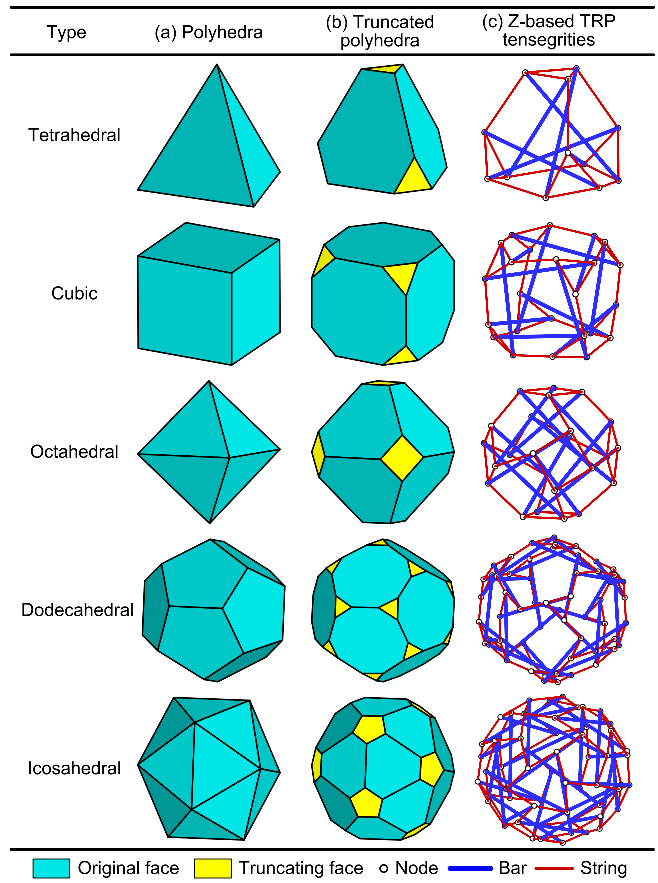

Fig. 1. Polyhedra and Z-based tensegrities: (a) regular polyhedra, (b) truncated regular polyhedra, and (c) Z-based TRP tensegrities.

Fig. 2(c). The Z-shaped cell highlighted in Fig. 2(b) has been replaced by the rhombic cell highlighted in Fig. 2(c).

However, it is emphasized that corresponding to the five types of Z-based TRP tensegrities, there are only three different types of rhombic TRP tensegrities for the following reasons. The rhombic truncated cubic and octahedral tensegrities have the same numbers of bars, strings and nodes, and the connection relations between the elements and the nodes are also identical. This indicates that both the topologies and connectivity matrices of a rhombic truncated cubic tensegrity and a rhombic truncated octahedral tensegrity can be expressed in the same form. Therefore, the rhombic truncated cubic and octahedral tensegrities can be regarded as the same type. For the same reasons, the rhombic truncated dodecahedral and icosahedral tensegrities can be incorporated into one type. The structural topologies of the three types of rhombic TRP tensegrities are shown in Fig. 4(a–c), which will be referred to as rhombic tetrahedral, cubic/octahedral, and dodecahedral/icosahedral tensegrities, respectively. According to the topology, the strings in a rhombic TRP tensegrity structure are classified into two types: type-1 and type-2. As can be seen from Fig. 4, each three type-1 strings form a triangle in all rhombic TRP tensegrities, while the type-2 strings form a triangle, a quadrangle, and a pentagon in rhombic truncated tetrahedral, cubic/octahedral, or dodecahedral/icosahedral tensegrities, respectively. A rhombic cell consists of one bar, two type-1 strings, and two type-2 strings, as shown in Fig. 2(c).

Referring to the Schläfli symbol $\{n, m\}$ for regular polyhedra, we further find that for all rhombic TRP tensegrities, the number of edges in a polygon consisting of type-1 strings, c, equals the smaller