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The micropolar elastic behaviour of model macroscopically heterogeneous materials

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ABSTRACT

This paper describes the design, manufacture, testing and analysis of two model heterogeneous materials that exhibit non classical elastic behaviour when loaded. In particular both materials demonstrate a size effect in which stiffness increases as test sample size reduces; an effect that is unrecognized by classical elasticity but predicted by more generalized elasticity theories that are thought to describe the behaviour of heterogeneous materials more fully. The size effect has been observed by both experimental testing and finite element analysis that fully incorporates the details of the underlying heterogeneity designed into each material. The size effect has been quantified thus enabling both the modulus and also the characteristic length, an additional constitutive parameter present within micropolar and other generalized elasticity theories, to be determined for each material. These characteristic length values are extraordinarily similar to the length scales associated with the structure of the materials. An additional constitutive parameter present within plane micropolar elasticity theory that quantifies shear stress asymmetry has also been determined for one of the materials by using an iterative process that seeks to minimize the differences between numerical predictions and test results.

1. Introduction

Most materials are usually heterogeneous; they are composed of an underlying structure that can be observed at some scale. Nevertheless, while the constitutive behaviour may vary from point to point this variation can be ignored because the size scale of the heterogeneity is insignificant; the overall behaviour can then be regarded as an averaging of any pointwise variation. This homogenization is assumed in constitutive theories such as classical elasticity (Sadd, 2005) which also assumes locality in that the state of stress at any point in a loaded material only depends on the state of strain there. However, when the scale of the underlying structure is comparable to the overall scale then the assumption of homogeneity may be invalid. This can occur in traditional materials such as ceramics and cement, more recently developed materials like particulate and short fibre reinforced composites and modern cellular and honeycomb materials (Gibson and Ashby, 1999) which are now being used ever more extensively because of the weight saving they afford. It can also arise in naturally occurring materials like wood, bone and rock and when supposedly homogeneous materials are used to manufacture micro mechanical systems. The consequences of material heterogeneity are believed to include size effects in which stiffness increases as overall size is reduced, modified elastic wave propagation and alteration of localized stress concentrations. More general theories of elasticity that account for material heterogeneity and predict these effects have been developed. These theories can be categorized into two broad classes; those that incorporate higher derivatives of displacement or strain gradients into the constitutive relations (Maugin and Metrikine, 2010) and those that incorporate additional degrees of freedom. The latter class includes Cosserat or micropolar elasticity (Sadd, 2005; Maugin and Metrikine, 2010; Eringen, 1966, 1999; Nowacki, 1972) in which rotational degrees of freedom that are independent of the conventional displacements are also included. These theories are inherently nonlocal and one of their common features is the inclusion of a length scale; an additional constitutive parameter that quantifies the size of the variation in the underlying structure and which must be measured by experiment. Other parameters may also have to be identified depending on the theory.

Experimental methods for determining the constitutive properties of expectedly micropolar materials based upon measuring size effects have been reported (Gauthier and Jahsman, 1975; Gauthier, 1981; Yang and Lakes, 1982; Lakes, 1983, 1986, 1995; Anderson and Lakes, 1994). Generally, these methods involve loading samples of material of similar geometry but varying size and identifying any variation in stiffness with size. Constitutive properties are then determined by comparing experimental observations of the size effect with analytical predictions for the particular loading mode employed. Identifying all the relevant parameters would usually involve testing in more than one loading mode, typically beam bending and torsion of rods. However, early attempts to identify size effects in a model material fabricated by encapsulating

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aluminium shot particles within an epoxy polymer matrix proved inconclusive (Gauthier, 1981); torsion test results produced considerable scatter and hinted that an anti size effect in which stiffness decreased with reducing sample size might be present. The inconsistency of this result has, in part, been explained by more recent theoretical work (Bigoni and Drugan, 2007) which indicates that for a heterogeneous material to exhibit micropolar behaviour the inclusions must be more compliant than the surrounding matrix and relatively dilute. Testing of real materials to identify their micropolar properties has also been conducted and results obtained for both polymeric foams (Lakes, 1986) and bone (Yang and Lakes, 1982). An elaborate experimental technique incorporating electromagnetic loading was employed to negate any local effects that mechanical loads would impose on the extremely small samples. Of the polymeric materials dense polyurethane foam appeared to exhibit micropolar behaviour while a syntactic or filled foam containing more rigid inclusions did not; an outcome that again concurs with theory. The need for careful sample preparation has also been reported (Anderson and Lakes, 1994); surface damage induced during the manufacture of closed cell polymethacrylimide samples was suggested as the cause of the observed size softening rather than the size stiffening anticipated. A method that identifies micropolar materials while negating the need to test samples of varying sizes has also been published (Lakes et al., 1985). This method relies on observing the opening of a crack located on the edge of a prismatic sample when twisted since crack opening will be displayed by a micropolar material but not by a homogeneous material.

In addition to the experimental effort that has been spent on attempting to identify the constitutive behaviour of heterogeneous materials considerable theoretical endeavour has also been expended in predicting behaviour by microstructural mechanics approaches. This work is particularly pertinent to understanding emerging micro and nano technological devices and structures. In general these endeavours attempt to represent materials at the microstructural level as some form of lattice structure that can then be represented by an assembly of individual elements. By considering an appropriate part of the assembly constitutive properties can then be estimated. Approaches of this type have been comprehensively reviewed previously (Ostoja-Starzewski, 2002).

The present paper examines the behaviour of another model micropolar material formed by introducing a repeated pattern of voids into an otherwise homogeneous material. The scale of the heterogeneity that the voids introduce was deliberately chosen so that the required range of material specimen sizes could all be loaded using commonplace mechanical testing equipment. In addition, the regularity of the void pattern facilitated finite element (FE) simulation of the material by using the element mesh to represent the solid matrix encapsulating the voids. This kind of detailed FE simulation affords the opportunity to investigate the effect of void distribution and volume fraction upon material behaviour without recourse to exhaustive experimentation.

Before describing the manufacture, testing and FE simulation of the model material a brief overview of micropolar elasticity is included for completeness. Simplifications of the general three dimensional constitutive equations are presented for both the planar case and also for that of a slender beam in bending.

2. Micropolar elasticity: an overview

In linear, three dimensional, micropolar elasticity the force stresses, τ_{ij}, and couple stresses, m_{ij}, are related to the deformations by (Eringen, 1966):

$$\tau_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + (2\mu^* + \kappa)\varepsilon_{ij} + \kappa e_{ijk}(\theta_k - \phi_k) \tag{1}$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \tag{2}$$

where the strain components, ε, are given in terms of the displacements, u, and microrotations, ϕ, by:

$$\varepsilon_{ij} = u_{j,i} + e_{jik}\phi_k \tag{3}$$

while θ are the conventional macrorotations, δ is the Kronecher delta symbol, e is the permutation tensor and the repeated indices denote summation over the range (i, j, k = 1, 2, 3). The six elastic constants, λ, μ^*, κ, α, β, and γ can be reinterpreted in terms of the engineering constants, E_M, G_M, υ_M, l_b, l_t, N and Ψ (Gauthier and Jahsman, 1975; Lakes, 1995) where the first three of these correspond to the Young's modulus, shear modulus and Poisson's ratio that govern uniform dilatational and distortional straining in a micropolar material as they do in classical elasticity. The subscript M is used here to distinguish them from their classical counterparts. The constants l_b and l_t are the characteristic lengths in bending and torsion respectively. In micropolar elasticity orthogonal shear stresses need not be complementary, any asymmetry being balanced by the couple stresses. The coupling number, N, then characterizes the asymmetry. The polar ratio, Ψ, plays a role akin to Poisson's ratio but relates orthogonal microrotations rather than dilatational strains.

In two dimensional Cartesian coordinates the strain displacement relations (3) can be expanded thus (Nakamura and Lakes, 1995):

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yx} \\ \varepsilon_{xy} \end{pmatrix} = \begin{pmatrix} u_{x,x} \\ u_{y,y} \\ u_{x,y} + \phi_z \\ u_{y,x} - \phi_z \end{pmatrix} \tag{4}$$

and if the membrane in a state of plane stress Eqs. (1) and (2) can be expressed as:

$$\begin{pmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{yx} \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} \frac{(2\lambda+2\mu^*+\kappa)(2\mu^*+\kappa)}{(\lambda+2\mu^*+\kappa)} & \frac{\lambda(2\mu^*+\kappa)}{(\lambda+2\mu^*+\kappa)} & 0 & 0 \\ \frac{\lambda(2\mu^*+\kappa)}{(\lambda+2\mu^*+\kappa)} & \frac{(2\lambda+2\mu^*+\kappa)(2\mu^*+\kappa)}{(\lambda+2\mu^*+\kappa)} & 0 & 0 \\ 0 & 0 & \mu^*+\kappa & \mu^* \\ 0 & 0 & \mu^* & \mu^*+\kappa \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{yx} \\ \varepsilon_{xy} \end{pmatrix} \tag{5}$$

and

$$\begin{pmatrix} m_{xz} \\ m_{yz} \end{pmatrix} = \begin{pmatrix} \gamma & 0 \\ 0 & \gamma \end{pmatrix} \begin{pmatrix} \phi_{z,x} \\ \phi_{z,y} \end{pmatrix} \tag{6}$$

or in terms of the engineering constants as:

$$\begin{pmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{yx} \\ \tau_{xy} \end{pmatrix} = \frac{E_M}{(1-v_M^2)} \begin{pmatrix} 1 & v_M & 0 & 0 \\ v_M & 1 & 0 & 0 \\ 0 & 0 & \frac{(1-v_M)}{2(1-N^2)} & \frac{(1-v_M)(1-2N^2)}{2(1-N^2)} \\ 0 & 0 & \frac{(1-v_M)(1-2N^2)}{2(1-N^2)} & \frac{(1-v_M)}{2(1-N^2)} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{yx} \\ \varepsilon_{xy} \end{pmatrix} \tag{7}$$

and

$$\begin{pmatrix} m_{xz} \\ m_{yz} \end{pmatrix} = \begin{pmatrix} \frac{2E_M l_b^2}{1+v_M} & 0 \\ 0 & \frac{2E_M l_b^2}{1+v_M} \end{pmatrix} \begin{pmatrix} \phi_{z,x} \\ \phi_{z,y} \end{pmatrix} \tag{8}$$

since in this case the out of plane force and couple stress components, τ_{xz}, τ_{yz}, τ_{zx}, τ_{zy}, τ_{zz}, m_{xy}, m_{yx}, m_{zx} and m_{zy} are all zero as are ϕ_x and ϕ_y, the microrotations about the x and y axes respectively.

Fig. 1 shows the components of force stress and couple stress acting on a differential element in the membrane. Static equilibrium of the element is described by two translational equilibrium equations as in classical elasticity: