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Pseudo-bistable pre-stressed morphing composite panels

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ABSTRACT

The pseudo-bistable phenomenon already shown to exist in the case of spherical domes is demonstrated in pre-stressed composite panels. This new concept for morphing structures uses intrinsic material viscoelasticity to actuate the structure passively between its different states. A pseudo-bistable structure is first snapped into a buckled state and allowed to relax under a constant strain. Once the actuation is removed, the structure remains in its buckled configuration for a period of time, before quickly returning to its initial state. In this paper, the principles of the pseudo-bistable behaviour are first outlined using a discrete truss model. An equivalent numerical model is then used to show how the time-dependent behaviour imparted to the structure can be controlled by the choice of the pre-straining boundary conditions. Next, the effect of a composite layup on the pseudo-bistable behaviour is shown, and a volume fraction limit is given. Finally, preliminary experimental results confirm the numerical simulations.

1. Introduction

Morphing structures in the form of adaptive surfaces offer exciting possibilities for aerospace applications. As opposed to complicated hinged joints and heavy mechanisms, morphing structures have the advantages of being lightweight, reliable, and containing no moving parts (Thill et al., 2007; Sofla et al., 2010). Actuation can be embedded into the structure, using actuators such as shape memory alloys and piezoelectric patches (Georges et al., 2009). Research on morphing structures has focused on composite materials to take advantage of their high stiffness-to-mass ratios and their long fatigue life. The intrinsic anisotropy of composite structures was first exploited by Schultz and Hyer (2003) using unsymmetric layups to produce bistable morphing structures. In their research, piezoceramic actuators were used to trigger the transition from one stable state to the other, and a Rayleigh–Ritz technique was used to obtain the stable states. More recently, the Rayleigh–Ritz method was employed by Pirrera et al. (2010) as a systematic approach to investigate the multistability of composite panels. The concept of bistability has also been extended to other configurations, such as pre-stressed bistable laminates (Daynes et al., 2008) and zero-stiffness twisting structures (Lachenal et al., 2012).

In this paper we aim to develop a new concept of self-actuating morphing structure using a combination of composite and viscoelastic materials. In conventional bistable structures, bilateral actuation is required to deform the structure from one state to the other. In this concept, however, only unilateral actuation is needed as the recovery of the structure to its initial state is self-actuated. In its undeformed configuration, the panel is monostable, and, after loading to its deformed state, the panel is allowed to relax under constant strain. Material viscoelasticity causes a change in the apparent stiffness of the structure, effectively causing the transition to a bistable structure. When the load is removed, the structure is able to remain in its second stable state for a determined period of time, before quickly snapping back to its original configuration. We say that such a structure is pseudo-bistable.

Previous work has concentrated on isotropic domes to understand the principles of pseudo-bistability (Santer, 2010; Brinkmeyer et al., 2012). However, it has been demonstrated, for example by the morphing scoop in Daynes et al. (2011), that double curvature is not necessary to achieve a bistable system. Building on this framework, the main objective of this research is to extend the pseudo-bistable behaviour to pre-stressed composite panels. In this paper, we first present the phenomenon of pseudo-bistability using a discrete model to explain its fundamental characteristics. Next, we show how the numerical model parameters influence the characteristic recovery time in the case of an isotropic panel. We then apply a composite layup to the panel and demonstrate the effect of volume fraction and layup asymmetry on the pseudo-bistable behaviour. Finally, the numerical model results are validated with simple experiments.

2. A discrete model of pseudo-bistability

2.1. Model definition

We first use an example of a discrete structure to demonstrate and define the principles of pseudo-bistability. Let us consider a

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modified form of the von Mises truss, which itself is a classic example of a bistable structure with one degree-of-freedom.

In elastic bistable structures, the load–displacement curve is independent of time and is generally characterised by three points where the external load is zero (Fig. 1). Points 1 and 3 are associated with a locally positive slope and define the stable equilibrium configuration of the structure. Point 1 is generally situated at the origin and represents the initial undeformed structure (trivial solution). Finally, point 2 has a locally negative slope and is unstable.

The structure can be transitioned between the different stable equilibrium configurations by applying an external force, so that at a critical value of the load P_{max} the structure snaps through, and when the load is released settles at point 3. A load P_{min} is needed for the structure to snap back to its original shape at point 1.

We now consider the discrete truss model in Fig. 2. The truss consists of two bars, connected by pin joints at the apex and at the base. Standard linear solid springs (linear viscoelastic springs) are introduced in the otherwise rigid bars (Lakes, 1999). Similarly, torsional viscoelastic springs simulate an elastic foundation at the base. The resulting linear and torsional spring stiffnesses at time t are respectively named $k_L(t)$ and $k_T(t)$. In addition, we will call: H, the initial height of the structure; w, the half-width; and z, the vertical displacement applied to point B. H and z are related to the initial angle α_0 and current angle α through

$$\alpha_0 = \tan^{-1}\left(\frac{H}{w}\right); \quad \alpha = \tan^{-1}\left(\frac{H-z}{w}\right). \tag{1}$$

Considering the equilibrium of the structure, the force in the bar AB is given by

$$P_{AB} = -k_L(L-L_0) = -k_L w\left(\frac{1}{\cos\alpha} - \frac{1}{\cos\alpha_0}\right). \tag{2}$$

Resolving vertically, the component of the reaction force from the linear spring is

$$P_L = P_{AB}\sin\alpha = -k_L w\left(\tan\alpha - \frac{\sin\alpha}{\cos\alpha_0}\right). \tag{3}$$

Similarly, the component of the reaction force from the torsional spring is

$$P_T = -\frac{k_T}{w}(\alpha - \alpha_0). \tag{4}$$

Finally, using Eqs. (3) and (4), the total vertical reaction force applied to the structure $P(t)$ is given in terms of the angle α by:

$$P(t) = -2\left[k_L(t)w\left(\tan\alpha - \frac{\sin\alpha}{\cos\alpha_0}\right) + \frac{k_T(t)}{w}(\alpha-\alpha_0)\right]. \tag{5}$$

The relaxation of the viscoelastic spring stiffnesses can be modelled by the following Prony series,

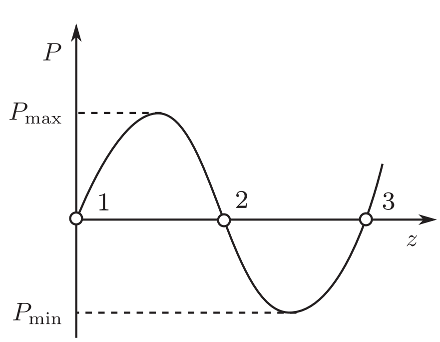

Fig. 1. Load–extension curve for a bistable structure. Points 1 and 3 are stable equilibrium points, whilst point 2 is unstable.

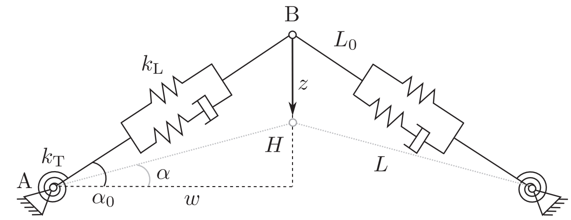

Fig. 2. Discrete truss structure with standard linear solid (SLS) springs. The torsional springs at the base also follow a SLS model but for clarity only the elastic torsional spring is shown.

$$k_L(t) = k_L(0)\left[1 - \sum_{i=1}^{N} k_{Li}\left(1 - e^{-t/\tau_i}\right)\right], \tag{6a}$$

$$k_T(t) = k_T(0)\left[1 - \sum_{i=1}^{N} k_{Ti}\left(1 - e^{-t/\tau_i}\right)\right]. \tag{6b}$$

where $k_L(0)$ and $k_T(0)$ are respectively the instantaneous linear and torsional stiffnesses, k_{Li} and k_{Ti} are respectively the linear and torsional relaxation coefficients, and τ_i are the relaxation times.

We also define $k^*(t)$ as the ratio between the linear and the torsional stiffness

$$k^*(t) = k_L(t)/k_T(t). \tag{7}$$

Finally, k^*_U is defined as the unrelaxed or instantaneous stiffness ratio, and k^*_R the relaxed or long-term stiffness ratio. In a relaxation test, i.e. applying an instantaneous strain to the structure and letting the structure relax for a time t_{rel}, the apparent stiffnesses decrease to $k_L(\infty)$ and $k_T(\infty)$ according to Eq. (6). Once the strain is removed, the stiffnesses, if measured directly, would increase or *recover* to the initial stiffnesses $k_L(0)$ and $k_T(0)$. This is associated with a recovery time t_{rec}. We assume that the recovery of the material is the reciprocal of the relaxation, so

$$k^*_U = k^*(t_{rel} = 0) = k^*(t_{rec} = \infty), \tag{8a}$$

$$k^*_R = k^*(t_{rel} = \infty) = k^*(t_{rec} = 0). \tag{8b}$$

In the purely elastic case, it can be shown that by decreasing the value of k^*, the position of the second stable equilibrium (point 3 in Fig. 1) is altered and the absolute value of the snap-back load P_{min} decreased. By continuity, there exists a critical ratio k^*_{crit} where the snap-back load P_{min} becomes zero. At this point, the non-trivial stable solution (point 3) disappears and the structure ceases to be bistable—it snaps back to its initial shape.

2.2. Numerical example

To illustrate this behaviour, we apply the following values to Eq. (5): $w = 50$ mm, $\alpha_0 = 20^\circ$ (which implies $H = 18.2$ mm), $k_L = 10$ N mm^{-1}, and $k_T = 410$ N mm, and plot the load response up to a maximum extension of $z_{max} = 35$ mm. The Prony series is defined in Table 1. Fig. 3a shows the evolution of the load–displacement curve with time.

Table 1
Prony series modelling the behaviour of the standard linear solid springs.

Term	k_{Li} [–]	k_{Ti} [–]	τ_i (s)
1	0.051	0.051	0.63
2	0.120	0.150	3.66
3	0.100	0.120	13.10
4	0.019	0.019	94.56