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Wave diffraction by a line of finite crack in a saturated two-phase medium

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ABSTRACT

An application of the Biot's theory to the diffraction problem of plane harmonic dilatational waves (P-waves) of the first kind and the second kind by a line crack or geometric discontinuity of finite width embedded in a saturated two-phase medium is presented in this paper. The crack surfaces are assumed impermeable, and the integral transform method is utilized to reduce the mixed boundary-value problem to a single Fredholm integral equation. The magnitudes of the intensity of the stress fields near the crack tips measured by Mode I dynamic stress-intensity factor (dimensionless) are computed and displayed graphically against dimensionless circular frequency (ω) for several dimensionless material property values, namely, viscosity-to-permeability and mass density ratios. In the case of the normally incident P-waves of the first kind, the results in terms of stress-intensity factor are also compared with the corresponding values of dry elastic material. All the stress-intensity factor curves are shown to exhibit a similar character in that they rise to the peaks at certain frequency values and then decay with increasing frequencies. At certain frequency ranges and material property values, amplification in the dynamic stress-intensity factor can be substantially larger than those encountered in dry elastic materials. The stress-intensity factor is found to be more affected by the changes in the ratio of viscosity-to-permeability at lower mass density ratio. With fluid mass density 10% of the bulk mass density, the viscosity-to-permeability ratio of 0.01 gives the highest increase of about 32% in the magnitude of stress-intensity factor compared to the dry material counterpart value, while a decrease of about 9% is observed for the viscosity-to-permeability ratio of 100. It is also found that change in mass density ratio has significant effect upon the magnitude of stress-intensity factor at lower ratio of viscosity-to-permeability. As for the normally incident P-waves of the second kind, the presence of the pore fluid affects both the magnitude and character of the stress-intensity factor. Large variations in the magnitude of stress-intensity factor are observed as viscosity-to-permeability ratio changes from 1 to 100. At the ratio of viscosity-to-permeability of 1.0, the stress-intensity factor curves increase gradually with frequency and exhibit the peaks in curves for mass density ratio of 0.3 and higher. As the viscosity-to-permeability ratio is raised to 100, the stress-intensity factor curves increase monotonically with frequency at a much faster rate throughout the frequency range of interest (ω = 0–2), and the change in mass density ratio is shown to have little effect on the stress-intensity factor, especially within the low frequency ranges. The results obtained in this study are useful in the mechanics of fracture initiation of saturated porous materials under the fluctuating mechanical and/or pore fluid loadings that are periodic with time.

1. Introduction

In dynamic fracture mechanics, the effects of fluctuating loads on elastic medium weakened by flaws or crack-like defects have been the focus of interest by many investigators, and the knowledge of the stress field around the crack tips is known to play a key role in determining the stability of the crack. On the other hand, systematic study is still needed to understand as to how the pore fluid and the fluctuating pore fluid loading can affect the local stress filed in saturated porous medium containing crack. Such knowledge can be practically important in many diversified fields, ranging from geomechanics to biomechanics.

For steady-state problems involving diffraction of harmonic waves by a finite crack in elastic medium, a considerable amount of research work has been devoted to the topic. Loeber and Sih (1968) provided the solution to the problem of diffraction of plane harmonic horizontally polarized shear waves (SH-waves) by a line crack of finite width, and presented a method of obtaining an asymptotically near-field solution. The method involves the use of the integral transform method to formulate the mixed boundary value problem, and gives the results in term of an auxiliary function governed by a Fredholm integral equation of the second kind. The same method was later extended by Sih and Loeber (1969a) to

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obtain the numerical results of an elastic medium with a line crack or geometric discontinuity of finite width disturbed by the propagation of plane harmonic P and SV-waves. Another attempt also has been made by the same authors, Sih and Loeber (1969b), utilizing the Hankel integral transform to solve the axisymmetric problem of scattering of normal compression and radial shear waves at a penny-shaped crack. A comprehensive review of the research work on the topic can be found in their paper. Other elastodynamic crack problems are also summarized by Sih (1968). Although most of the studies on the topic have been progressed considerably, but they are still limited to elastic medium with a single solid constituent or dry elastic medium.

The purpose of this paper is to investigate the response of a saturated porous elastic medium containing a line crack of finite width under the propagating plane harmonic dilatational waves (also referred to as compressional waves, or P-waves) of the first kind and the second kind generated, respectively, by fluctuating mechanical and/or pore fluid loadings. Of particular interest is the influence of certain material properties, namely, mass density of the fluid relative to bulk mass density and viscosity-to-permeability ratio, upon the intensity of the stress fields in the vicinity of the crack tips. The formulation of the problem is based on the linear theory of Biot (1962) (also known as the Biot's theory) which takes into account the interdependence between the two constituents, i.e., fluid and solid skeleton, and is basically analogous to the linear theory of coupled thermoelasticity as pointed out by Biot (1956). Reference to the topic concerning the transient problem of thermal loadings suddenly applied to the crack surfaces can be made to the work of Kassir et al. (1986). The diffraction problem of plane harmonic, dilatational and thermally induced thermoelastic waves interrupted by a crack of finite width in an unbounded elastic medium has been solved by Phurkhao and Kassir (1991). They assumed that the crack surfaces are insulated, and formulated the problem utilizing the integral transforms to reduce the problem to a single Fredholm integral equation of the second kind, and then obtained the results in terms of the stress-intensity factor (k_1). Their solution method will be extended to solve a similar problem relating to a fluid-saturated porous material in this paper.

In regard to embedded crack in an unbounded, saturated, porous medium subjected to time dependent loadings, few problems have been solved in the past. Craster and Atkinson (1996) have attempted to find the asymptotic solution of the transient problem of finite crack in a poroelastic medium using matched asymptotic expansions and rescalings, but their solution is based on the quasi-static theory of Biot (1941) in which all the inertia terms are neglected. Additional references on the theory should also be made to the work of Rice and Cleary (1976). In a recent paper, Jin and Zhong (2002) obtained a transient solution of an axisymmetric problem of an impermeable penny-shaped crack embedded in an infinite porous solid under the suddenly applied uniform traction over the crack surfaces. The problem of diffraction of harmonic dilatational wave of the first kind incident upon a permeable circular crack in a saturated poroelastic medium has also been treated by Galvin and Gurevich (2007) using the Hankel transform to reduce the problem to a Fredholm integral equation. However, their concerns are not in the stress field around the crack tips, and the response of the medium under the incidence of the P- waves of the second kind has not been investigated. To the best of the author's knowledge, there are no analytical solutions to the Biot's general field equations of the diffraction problem of plane harmonic dilatational waves obstructed by a line crack of finite width within an unbounded medium.

Section 2 of this paper describes the basic equations governing the wave propagation in the two-phase medium, and begins with the introduction of the governing equations of motion of the bulk material coupled with the diffusion equation of the fluid in the non-dimensional space and time variables. The equations are then decomposed into the dilatational and shear wave equations. Section 3 considers two types of incident waves, namely, mechanically induced P-waves of the first kind and the P-waves of the second kind induced by the pore fluid pressure. Both input waves are treated separately in subsequent analysis to determine solutions of the scattered wave fields. Section 4 deals with the formulation and the determination of the scattered wave fields utilizing the integral transform technique as outlined by Phurkhao and Kassir (1991) to reduce the mixed boundary-value problem to a pair of dual integral equations whose solution is in turn governed by a Fredholm integral equation of the second kind which is suitable for numerical work. Finally, numerical results of the two-phase medium computed as the magnitude of dimensionless stress-intensity factors are presented graphically in Section 5 for several values of mass density and viscosity-to-permeability ratios. In the case of incident P-waves of the first kind, the results are also compared with dry material counterpart values over a range of dimensionless frequency of interest.

2. Basic equations

Consider a through crack of finite width, $2a$, embedded in an infinite medium consisting of a porous elastic solid saturated by a viscous fluid (two-phase material). With reference to the Cartesian coordinates (x', y', z') and time t', it is assumed that the crack is subjected to the plane harmonic waves propagating in the medium in the $x'y'$-plane. In the absence of the body forces, the basic equations governing the motion of the two-phase medium based on the Biot's theory (1962) are

$$\tau'_{ij,j} = \rho \ddot{u}'_i + \rho_f \ddot{w}'_i, \tag{1}$$

$$-p'_{,i} = \rho_f \ddot{u}'_i + \frac{\rho_f}{f} \ddot{w}'_i + \frac{\eta}{k} \dot{w}'_i. \tag{2}$$

Here, $u'_j,\ j = x, y, z$ denote the components of the displacement vector $\mathbf{u}' = (u'_x,\ u'_y,\ u'_z)$ of the solid; $w'_j = f(U'_j - u'_j),\ j = x, y, z$ represent the components of the fluid displacement vector relative to the solid portion $\mathbf{w}' = (w'_x,\ w'_y,\ w'_z) = f(\mathbf{U}' - \mathbf{u}')$, with f being the porosity of the medium. Also, $\rho_f,\ \rho$ designate, respectively, the mass density of the fluid and the bulk material, while, $\eta,\ k$ are the fluid viscosity and permeability (absolute) of the medium, respectively. Moreover, a subscript comma is used to denote partial differentiation, while a dot over a field variable indicates differentiation with respect to the time variable. The total or bulk stress components $\tau'_{ij},\ (i, j = x, y, z)$ and the pore pressure of the fluid p' are related to the displacement components through the following constitutive equations

$$\tau'_{ij} = 2\mu\varepsilon'_{ij} + \delta_{ij}\lambda e - \delta_{ij}\alpha p', \tag{3}$$

$$p' = M(-\alpha e + e_f), \tag{4}$$

in which, $\varepsilon'_{ij},\ (i, j = x, y, z)$ denote the components of the strain tensor, δ_{ij} stands for the usual Kronecker delta symbol, and e designates the dilatation of the solid skeleton, i.e.,

$$\varepsilon'_{ij} = (u'_{i,j} + u'_{j,i})/2, \tag{5}$$

$$e = u'_{j,j} \tag{6}$$

Moreover, the dilatation of the fluid portion measuring the variation in the fluid content is given by

$$e_f = -w'_{j,j}. \tag{7}$$

The constants $\alpha,\ M$ are, respectively, referred to as the Biot coefficient and the Biot modulus. In addition, the shear modulus of the bulk material is represented by μ, and λ is the Lame's constant of the bulk material under the constant pore pressure (or open system)