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Zeroth-order shear deformation micro-mechanical model for composite plates with in-plane heterogeneity

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A R T I C L E   I N F O

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A B S T R A C T

This article introduces a new model for investigating the mechanical behavior of heterogeneous plates, which are composed of periodically-repeated microstructures along the in-plane directions. We first formulate the original three-dimensional problem in an intrinsic form for implementation into a single unified formulation and application to geometrically nonlinear problem. Taking advantage of smallness of the plate thickness-to-length parameter and heterogeneity and performing homogenization along dimensional reduction simultaneously, the variational asymptotic method is used to rigorously construct an effective zeroth-order plate model, which is similar to a generalized Reissner–Mindlin model (the first-order shear deformation model) capable of capturing the transverse shear deformations, but still carries out the zeroth-order approximation which can maximize simplicity and promote efficiency. This present approach is incorporated into a commercial analysis package for the purpose of dealing with realistic and complex geometries and constituent materials at the microscopic level. A few examples available in literature are used to demonstrate the consistence and efficiency of this new model, especially for the structures, in which the effects of transverse shear deformations are significant.

1. Introduction

Composite materials have demonstrated excellent potential for use as high-strength and low-weight materials, superior noise and energy absorption ones, and high-temperature resistance ones. Due to these characteristics suitable for and these rapidly increasing popularity in various engineering applications, research attentions devoted to accurate and general modeling for predicting and controlling their properties have received considerable attention in the past several decades. Moreover, board understandings and elaborate fabrication techniques of them are even possible to manufacture new microstructure-based materials and structures to achieve the ever-increasing performance requirements. Although the full three-dimensional (3D) finite element analysis (FEA) is widely accepted and used for analysis of such materials and structures by meshing all the details of constituent microstructures, it is not an efficient and convenient way because of the inordinate requirements of computational cost to capture the micro-scale mechanical characteristics.

For this reason, there is clearly a need for an alternative approach of the full 3D FEA, especially, by using a unit cell (UC). It allows for a practical definition of the building block of the heterogeneous material, and leads to replacing the original heterogeneous structure with a homogeneous one with a set of effective material properties if the size of UC (d) is much smaller than the size of the structure (L) (i.e. $\eta = d/L \ll 1$). See Hollister and Kikuchi (1992), Kalamkarov et al., 2009 and Kanouté et al., 2009 for a review. However, since most of these approaches consider the heterogeneous structure as a periodic assembly of many UCs along the whole direction, those are not suitable for engineering analyses of dimensionally reducible structures (Lewiński, 1991; Buannic and Cartraud, 2001; Yu, 2002; Lee and Yu, 2011), i.e. those with one or two dimensions much smaller than others and made of a finite number of UCs along the smaller ones. For example, many load-bearing components are flat panels with the thickness h much smaller than the in-plane dimensions L (i.e. $e = h/L \ll 1$), and they can be effectively modeled using the plate theory. Note that, here and throughout the paper, the full (or direct) 3D FEA refers to the standard 3D structural modeling by meshing all the details of constituent microstructures with 3D brick elements.

In recent years, the formal asymptotic method (Caillerie, 1984; Kohn and Vogelius, 1984; Lewiński, 1991; Parton and Kudryavtsev, 1993; Buannic et al., 2003; Kalamkarov et al., 2009) and the computational homogenization method (Geers et al., 2007; Coenen et al., 2010) have been applied to study this problem. Both methods are a direct application of the formalism of two scales to the original 3D equations governing the plate structure. First, the formal asymptotic method is a modification to the asymptotic homogenization method. Although it is mathematically elegant

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and rigorous in developing a simple engineering model (e.g. the Love–Kirchhoff plate model), it is not easy to extend this approach to yield a refined theory (e.g. the Reissner–Mindlin model) without the help of the alternative and indirect way to predict the transverse shear stiffness, such as performance of the simply supported beam made of several UCs (Buannic et al., 2003; Sharma et al., 2010). Last but not least, it is difficult to implement such procedures numerically. On the other hand, the computational homogenization scheme is one of the most accurate numerical techniques in determining the local governing behavior of a well-characterized microstructure through solving the macro-scale and micro-scale boundary problems simultaneously. However, it is still limited to the simple engineering model under the assumption of the Love–Kirchhoff shell theory.

As a remedy to the shortcomings of formal asymptotic methods and computational homogenization methods, we propose to use the variational asymptotic method (VAM) developed by Berdichevsky (1979) to carry out simultaneous homogenization and dimensional reduction, to construct an engineering model suitable for plates made of heterogeneous materials, and to extend our previous works (Lee and Yu, 2011; Lee et al., in press) by producing a refined model that includes transverse shear effects, but still carries out the zeroth-order approximation which can maximize simplicity and promote efficiency. The VAM is a mathematical approach, which includes the merits of both variational (simplicity and brevity) and asymptotic (no ad hoc kinematic assumptions) methods, and can be used to solve any problem governed by an energy functional having one or more small parameters inherent to the structure (Berdichevsky, 2009). First, under the concept of decomposition of the rotation tensor (Danielson and Hodges, 1987), the 3D anisotropic elasticity problem is formulated in an intrinsic form suitable for geometrically nonlinear analysis. The procedure is both general and accurate for allowing all possible forms of deformation, while also being simple and compact for avoiding a tediously and near-intractably displacement-based representation in terms of displacement and rotation variables. Then, considering both smallness of the plate thickness-to-length parameter (e) and heterogeneity (η), we use VAM to rigorously decouple the original 3D anisotropic, heterogeneous problem into (i) a linear two-dimensional (2D)/3D micro-mechanical analysis under only distributed applied loads at the top and bottom surfaces and the whole body force, such as an uniform pressure, and (ii) a nonlinear 2D macro-surface analysis (i.e. plate analysis) subject to various in-plane boundary conditions on the plate edges (e.g. simply supported, clamped, or free), applied loads at the top and bottom surfaces, and body forces on the whole plate. The micro-mechanical analysis can be implemented in COMSOL MULTIPHYSICS™ (COMSOL), a finite element based simulation and modeling tool. Whatever tool is used, it must be sufficiently general to numerically solve the partial differential equations (PDEs) that one frequently encounters when obtaining the elastic constants for a refined 2D plate analysis and the recovery relations that yield local displacement, strain, and stress fields based on the macroscopic behavior. Several examples, in which the transverse shear deformation is especially significant, are used to demonstrate the application and accuracy of this new model and the companion code.

2. Plate kinematics and 3D formulation with homogenization

When a 3D elastic body has one dimension much smaller than the other two, it can be geometrically treated as a plate (ω), a smooth 2D reference plane s surrounded by material with thickness h. In general, a point in the plate can be represented mathematically by its Cartesian coordinates x_i , where x_α are two orthogonal lines in the reference plane while x_3 is the normal coordinate in the undeformed plate. (Here and throughout the paper, Greek indices assume values 1 and 2 while Latin indices assume 1, 2, and 3. Repeated indices are summed over their range except where explicitly indicated.) Without loss of generality, we choose the middle surface of the plate as the origin of x_3. Let us now consider an heterogeneous plate composed of periodically-repeated unit cells (UCs) denoted by Ωs in the reference plane; see Fig. 1. To implement the homogenization procedure into the present approach, and then to describe the rapid change in the material characteristics along the in-plane directions, we need to introduce the so-called "fast" variations y_α parallel to the "slow" variations x_α. These two sets of variations are related as $y_\alpha = x_\alpha/\eta$.

If the UC is depicted in Fig. 1, we can describe the domain (Ω) occupied by the UC using y_α and $y_3 = x_3$ as

$$\Omega = \left\{ (y_1, y_2, x_3) \middle| -\frac{d_1}{2} < y_1 < \frac{d_1}{2}, -\frac{d_2}{2} < y_2 < \frac{d_2}{2}, -\frac{h}{2} < x_3 < \frac{h}{2} \right\} \tag{1}$$

In order to homogenize the heterogenous material characteristics in the plate, we assume that the exact solution of the field variables have volume averages over Ω. For example, if $u_i(x_1, x_2, x_3; y_1, y_2)$ are the exact displacements within the UC, there exists the corresponding effective ones $v_i(x_1, x_2)$ such that

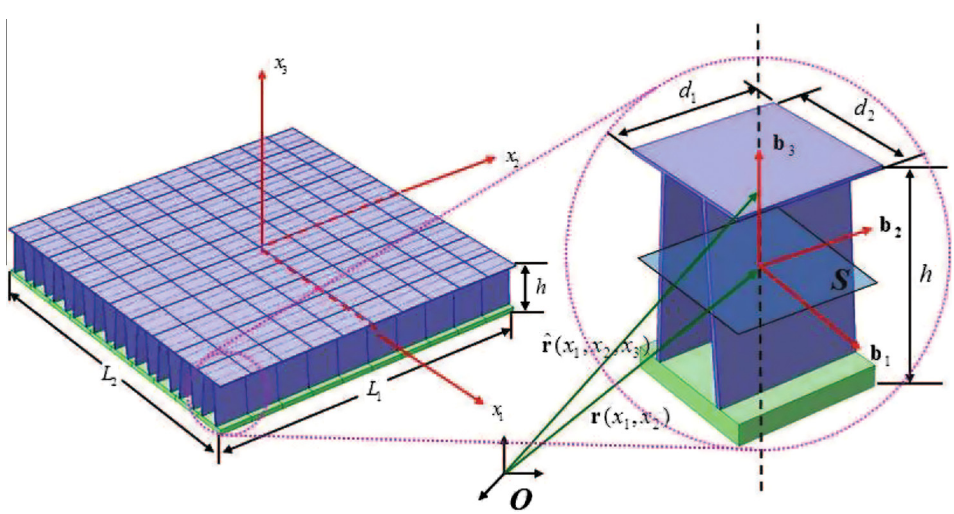

Fig. 1. A heterogeneous plate with periodically-repeated unit cell (Lee et al., in press).