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A nonuniform TFA homogenization technique based on piecewise interpolation functions of the inelastic field

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ABSTRACT

The present paper deals with a homogenization technique based on the Transformation Field Analysis (TFA) for the study of heterogeneous composite media characterized by nonlinear response. According to the TFA, the behavior of the representative volume element (RVE) is studied accounting for the nonlinear effects by means of the presence of a uniform inelastic strain distribution in the nonlinear constituent of the heterogeneous material. In order to improve the TFA, the assumption of uniformity of the inelastic strain distribution is removed, so that a nonuniform inelastic strain field, better representing the inelasticity distribution in the composite, is considered. In particular, the inelastic strain is represented as a piecewise linear combination of analytical functions of the spatial variable. The theory, presented in a general framework, can be successfully adopted for deriving the overall constitutive response for a wide range of nonlinear composite materials. Furthermore, the procedure is tailored to investigate the response of composites whose constituents are Shape Memory Alloys (SMA) and materials characterized by plastic behavior. Finally, numerical applications are developed in order to assess the effectiveness of the proposed nonuniform TFA procedure, comparing the results with the ones carried out performing Uniform and Piecewise Uniform TFA homogenizations and nonlinear finite element micromechanical analyses.

1. Introduction

The increasing use of composite materials for high performance applications in several fields of structural engineering has brought out the need for the development of efficient tools able to provide suitable numerical predictions of their mechanical response. Accordingly, numerous homogenization techniques, which allow to estimate the mechanical properties of heterogeneous media and to derive the overall behavior of the equivalent homogenized material, have been developed. Many homogenization procedures available in literature are based on various effective medium models such as the equivalent eigenstrain method proposed by Eshelby (1957), the Mori and Tanaka approach (Mori and Tanaka, 1973), which used the Eshelby solution, the self-consistent model of Hill (1965), the variational approaches of Hashin and Shtrikman, leading to their well-known bounds (Hashin and Shtrikman, 1963), which have been generalized by Willis (1977, 1983), among many others.

All the mentioned homogenization techniques, initially formulated to study the linear response of the heterogeneous material, represent the basis for the development of suitable techniques able to consider nonlinear material effects. Of course, the determination of the mechanical behavior of a composite becomes a more complex process, when its constituents are characterized by inelastic phenomena.

A branch of homogenization has focused on the introduction of rigorous bounds and estimates for the effective properties of nonlinear composites, using variational methods. Talbot and Willis (1985) obtained rigorous bounds through the generalization to nonlinear media of the Hashin–Shtrikman variational principle. Ponte Castañeda (1991) proposed an alternative variational structure able to generate bounds and estimates for nonlinear composites from the corresponding information of linear composites with the same microstructural distribution, and he also provided estimates exact to second order in the heterogeneity contrast (Ponte Castañeda, 1996). Lately, Agoras and Ponte Castañeda (2011) applied the method proposed in Ponte Castañeda (1991) to multiscale composites with viscoplastic isotropic constituents and random sub-structures.

In order to accurately describe the response of a nonlinear heterogeneous medium, numerical techniques can be adopted to solve the micromechanical problem. In fact, the finite element method or the boundary element method, can be successfully adopted; on the other hand, these numerical techniques involve a large number of internal variables and, thus, lead to high computational burden.

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Simplified homogenization approaches have been proposed with the aim of reducing the computational complexity of the numerical micromechanical investigation.

The micromechanical method of cells (Aboudi, 1991), and later its generalization (Aboudi, 1996), was proposed by Aboudi with the aim of providing the overall behavior of periodic multiphase materials of various types.

An interesting approach for solving the nonlinear micromechanical homogenization problem is the Transformation Field Analysis (TFA), originally proposed by Dvorak (1992). It considers the inelastic strain as a given eigenstrain, assuming the field of internal variables to be uniform in each individual constituent of the composite characterized by nonlinear behavior. The effect of eigenstrains is accounted for by solving linear elasticity problems and it is superimposed to the effect induced by uniform overall strain. In particular, for two-phase composites the TFA method is able to provide exact relations between the elastic strain concentration tensors and the influence tensors due to the Levin-Rosen-Hashin's formula (Dvorak and Bahei-El-Din, 1997).

The TFA method was also improved by Dvorak et al. (1994) through the subdivision of each material phase into several subsets, developing a PieceWise Uniform TFA (PWUTFA). Considering a piecewise uniform inelastic strain distribution enables to improve the description of the inelastic strain heterogeneities but, obviously, at the expense of increasing the complexity of the technique. However, it has been shown that the TFA and PWUTFA techniques lead to too stiff predictions (Suquet, 1997). Chaboche et al. (2001) also developed a PWUTFA in order to derive the nonlinear behavior of damaging composites. In order to better represent the complex field of the inelastic strain in the representative volume element (RVE) of a nonlinear composite, Michel and Suquet (2003) proposed a nonuniform TFA (NUTFA). The inelastic strain field is considered as nonuniform and described as the superposition of functions, called inelastic modes and determined numerically by simulating the response of the composite along monotone loading paths. The technique was then implemented for the study of nonlinear composite materials (Michel and Suquet, 2004). Lahellec and Suquet (2007) proposed an alternative method for deriving the overall properties of nonlinear inelastic composites based on the minimization of an incremental energy function, within an implicit time-discretization scheme; they proved that this approach is equivalent to the transformation field analysis with a nonuniform eigenstrain field (Michel and Suquet, 2003). Recently, the nonuniform TFA model (Michel and Suquet, 2003) has been usefully applied by Franciosi and Berbenni (2007, 2008) for modeling heterogeneous crystal and poly-crystal plasticity, characterized by hierarchical multi-laminate structures; by Roussette et al. (2009) for the study of composites having elastic-viscoplastic and porous elastic-viscoplastic constituents; by Fritzen and Böhlke (2011) for the analysis of the morphological anisotropy of micro-heterogeneous materials with particle reinforcement; by Jiang et al. (2011) for the analyses of nonlinear composite media made of porous materials.

Marfia and Sacco (2005), Marfia (2005), and Sacco (2009) developed TFA and PWUTFA homogenization procedures which consider the periodicity conditions in order to investigate the overall nonlinear response of the Shape Memory Alloy (SMA) composites and masonry materials. Marfia and Sacco (2007) also proposed a multiscale approach for SMA composite laminates developing a nonlinear homogenization technique. Addessi et al. (2010) have extended the PWUTFA technique to the Cosserat continuum to study the mechanical response of masonry. Marfia and Sacco (2012) presented a PWUTFA homogenization procedure for the multiscale analysis of periodic masonry, assuming a bilinear approximation for the inelastic strain of one subset of the unit cell.

Aim of the present paper is the development of a new homogenization technique for composite materials characterized by the nonlinear behavior of their constituents in the framework of the transformation field analysis. Following the proposal discussed in Michel and Suquet (2003) and in Marfia and Sacco (2012), the field of the internal variables is considered as nonuniform. The main novelties of this work consist in the construction of an approximated nonuniform inelastic strain field in the RVE of the composite medium and in the derivation of its evolutive problem. The RVE is divided into subsets; in each subset a nonuniform distribution of the inelastic strain, which accounts for all the nonlinear effects, is adopted. In particular, the inelastic strain in each subset is assumed as a linear combination of selected analytical functions, called modes, which depend on the spatial variable. The coefficients of the linear combination are tensors and are determined solving the evolutive problem. In particular, a new formulation is proposed to compute the evolution of the coefficients of the approximated form of the inelastic strain on the basis of the continuum evolutive equations.

In the present contribution numerical applications are developed considering different types of periodic composite materials. In particular, plasticity and shape memory alloy models are implemented in order to take into account the nonlinear phenomena occurring in the material constituents of the examined composites.

In order to verify the efficiency of the developed procedure, the numerical results obtained by the proposed nonuniform TFA homogenization technique are compared with the ones carried out adopting uniform and piecewise uniform TFA procedures and nonlinear finite element micromechanical analyses.

The paper is organized as follows: Section 2 formulates the problem of the homogenization for nonlinear composites; in Section 3, the formulation of the proposed nonuniform TFA procedure is presented; in Section 4, the developed nonuniform TFA is tailored for plastic and SMA materials, illustrating the numerical procedure; Section 5 deals with the numerical applications.

2. Homogenization problem for nonlinear composites

The homogenization problem for composite media whose constituents can be characterized by nonlinear response, is studied in the framework of small strain theory. In particular, the interest is devoted to inelastic phenomena induced by plasticity, damage, visco-plasticity and other nonlinear effects.

The mathematical algebraic notations adopted in the following are briefly introduced. Scalar variables are denoted in italics; vectors, second-order tensors and fourth-order tensors in bold; scalar product between vectors with symbol '·'; scalar product between two second-order tensors with symbol ':'; product between two fourth-order tensors, a fourth-order tensor and a second-order tensor, a second-order tensor and a vector (with the relative contractions of two indices, two indices and one index, respectively) with no symbol.

A general representative volume element, able to retain the properties of the heterogeneous medium characterized by nonlinear behavior, is considered. In the following the typical RVE is denoted with the symbol Ω.

The constitutive relationships for the constituents of Ω are formulated in a phenomenological thermodynamic framework (Halphen and Nguyen, 1975). In fact, the existence of a thermodynamic potential is postulated and a free specific energy function is introduced through a convex potential as:

$$\Psi(\boldsymbol{\varepsilon}, \boldsymbol{\pi}, T, \mathbf{V}_k) = \Psi_e(\boldsymbol{\varepsilon}, \boldsymbol{\pi}, T) + \Psi_p(\boldsymbol{\pi}, \mathbf{V}_k, T), \tag{1}$$