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Supersonic responses induced by point load moving steadily on an anisotropic half-plane

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ABSTRACT

Supersonic responses of an anisotropic half-plane solid induced by a point load moving steadily on the half-plane boundary are investigated. Analytic expressions for the responses of the displacements and stresses for field points either inside or on the surface of the half-plane solid are given for general anisotropic materials. For the special cases of monoclinic materials with symmetry plane at $x_3 = 0$ and orthotropic materials, the supersonic as well as subsonic responses of the displacements and stresses are further expressed explicitly in terms of elastic stiffnesses. Responses for the case of isotropic materials known in the literature are recoverable from present results.

1. Introduction

The responses of a linearly elastic *isotropic* solid induced by loads moving steadily on the half-plane boundary were classified into three different cases, namely, subsonic, transonic and supersonic cases, depending on how fast of the speed v of the loads is. The two wave speeds $v_L = \sqrt{(\lambda + 2\mu)/\rho}$ and $v_T = \sqrt{\mu/\rho}$, where λ and μ are the Lame constants and ρ is the mass density, play the role in the classification. For the speed $v < v_T$, the case is called subsonic while for the speed $v > v_L$ we have the case of supersonic. The transonic case corresponds to the speed falling in the range $v_T < v < v_L$. All of these three cases are of considerable importance in many branches of applications and have been investigated by many researchers (Barber, 1996; Cole and Huth, 1958; Eason, 1965; Eringen and Suhubi, 1975; Georgiadis and Barber, 1993; Georgiadis and Lykotrafitis, 2001; Rahman, 2001; Sneddon, 1952). Their analyses are all for isotropic materials.

The classification of *anisotropic* solid induced by loads moving steadily on the half-plane boundary depends on the occurrence of the roots of the Stroh eigenvalue problem for steady state problem (Chadwick and Wilson, 1992; Lothe and Alshits, 2009; Ting and Barnett, 1997). The Stroh eigenvalue problem (see Eq. (2.4)) provides six p_α which is real or complex conjugate pairs. For $\boldsymbol{v} = 0$, the problem corresponds to elastostatics where the six eigenvalues p_α are all complex and they appear in pairs of complex conjugates. As v increases from zero, one or more pairs of eigenvalues p_α will become real. The speed $v = \hat{v}$ at which one pair of the p_α first becomes real is defined as the limiting wave speed (Lothe and Alshits, 2009; Ting, 1996). When the speed v of the loads falls in the range $0 < v < \hat{v}$ the problem is called *subsonic* problem, while for $v > \hat{v}$ the corresponding problem is called *supersonic* problem. The problem occurred exactly at limiting speed, i.e., at $v = \hat{v}$, is called *transonic* problem. There are other *transonic* problems occurred for $v > \hat{v}$. These problems embody those real eigenvalues p_α or which they are not distinct. Note that the terminology "transonic" adopted for classifying the responses for anisotropic materials is different from that for isotropic materials. The subsonic problem where the six eigenvalues p_α all remain complex have been dealt with by Liou and Sung (2008). Their analyses focus on the surface responses (Liou and Sung, 2008) for point load or uniform traction moving steadily on an anisotropic half-plane boundary. In this paper, the supersonic responses of a half-plane anisotropic solid due to a steadily moving point load are investigated, for field points either inside or on the surface of the half-plane solid. There are three cases (called S1, S2 and S3 cases in the context) needed to be distinguished for supersonic problem, depending on how many pairs of eigenvalues being real occurred in the supersonic problem. For all cases studied, the responses of displacements and stress components are presented in closed forms for general anisotropic materials. The obtained analytic expressions for general anisotropic materials are further exploited for monoclinic materials with symmetry plane at $x_3 = 0$ and for orthotropic materials, not only for supersonic problem but also

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for subsonic problem. For these two special materials, the exploited displacements are found to be related to matrix $\boldsymbol{\Lambda}(v)$ while those for stress components are related to matrices $\boldsymbol{\Omega}(v)$ and $\boldsymbol{\Gamma}(v)$. All elements of the matrices $\boldsymbol{\Lambda}(v)$, $\boldsymbol{\Omega}(v)$and $\boldsymbol{\Gamma}(v)$ are composed by terms involving the field points of observations multiplied by parameters which are completely determined by the elastic stiffness as well as the speed v. As the field points of observations are moved to the half-plane boundary, results for subsonic problem have been verified with those presented by Liou and Sung (2008). The multiplied parameters given analytically are also studied numerically for a selected monoclinic material. By taking special values in the analytic expressions for the elements of the matrices $\boldsymbol{\Lambda}(v)$, $\boldsymbol{\Omega}(v)$ and $\boldsymbol{\Gamma}(v)$, the results for isotropic materials appeared in the literature, both for subsonic and supersonic problems, are all recovered.

2. Basic equations

Consider a linearly elastic half-plane solid subjected to a point load moving steadily with speed $v > 0$ over its surface (Fig. 1). Let a Cartesian coordinate x_1, x_2, x_3 move with the load. Then viewed from this moving coordinate system, the steady state responses of the displacement $\boldsymbol{u} = [u_1, u_2, u_3]^T$ are governed by

$$(\boldsymbol{Q} - \rho v^2 \boldsymbol{I})\boldsymbol{u}_{,11} + (\boldsymbol{R} + \boldsymbol{R}^T)\boldsymbol{u}_{,12} + \boldsymbol{T}\boldsymbol{u}_{,22} = \boldsymbol{0}, \quad (2.1)$$

where ρ is the mass density, $\boldsymbol{I}$ is a 3×3 unit real matrix, the superscript T stands for the transpose, a comma indicates partial differentiation, and

$$\boldsymbol{Q} = \begin{bmatrix} c_{11} & c_{16} & c_{15} \\ c_{16} & c_{66} & c_{56} \\ c_{15} & c_{56} & c_{55} \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} c_{16} & c_{12} & c_{14} \\ c_{66} & c_{26} & c_{46} \\ c_{56} & c_{25} & c_{45} \end{bmatrix},$$
$$\boldsymbol{T} = \begin{bmatrix} c_{66} & c_{26} & c_{46} \\ c_{26} & c_{22} & c_{24} \\ c_{46} & c_{24} & c_{44} \end{bmatrix}. \quad (2.2)$$

Here all the elements of $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{T}$, determined only by the material constants, are expressed in terms of $c_{\alpha\beta}(\alpha, \beta = 1,2,4,5,6)$ which are the contracted notations of the elastic stiffness c_{ijks}. Both $\boldsymbol{Q}$ and $\boldsymbol{T}$ possess the symmetric and positive definite properties. The solution to Eq. (2.1) can be expressed as:

$$u_i = a_i\, g(z), \quad \text{or} \quad \boldsymbol{u} = \boldsymbol{a}\, g(z), \quad (2.3)$$

where $z = x_1 + px_2$, g is an arbitrary function of z. Unknown constant p and vector $\boldsymbol{a}$ are determined by the eigenrelation

$$\boldsymbol{U}\boldsymbol{a} = \boldsymbol{0}, \quad (2.4)$$

where

$$\boldsymbol{U} = [(\boldsymbol{Q} - \rho v^2 \boldsymbol{I}) + p(\boldsymbol{R} + \boldsymbol{R}^T) + p^2 \boldsymbol{T}]. \quad (2.5)$$

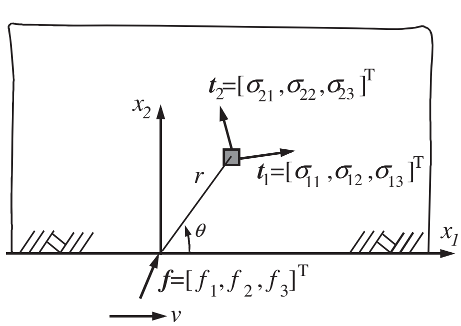

Fig. 1. Loads moving steadily over the anisotropic elastic half-plane surface.

The eigenrelation (2.4) provides six eigenvalues p_α and associated six eigenvectors $\boldsymbol{a}_\alpha(\alpha = 1–6)$. The dependence of p_α and $\boldsymbol{a}_\alpha(\alpha = 1–6)$ on v is implied implicitly. Here and in what follows, we assume that the six eigenvalues are all *distinct*. Therefore, by superposing the six independent solutions associated with the six distinct eigenvalues, the general solution for the displacement may be obtained as follows (Ting, 1996)

$$\boldsymbol{u} = \sum_{\alpha=1}^{6} \boldsymbol{a}_\alpha g_\alpha(z_\alpha) \quad \text{or} \quad \boldsymbol{u} = \boldsymbol{A}\boldsymbol{g}(\boldsymbol{z}) + \hat{\boldsymbol{A}}\hat{\boldsymbol{g}}(\hat{\boldsymbol{z}}), \quad (2.6)$$

where

$$\begin{aligned} &\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3], \quad \hat{\boldsymbol{A}} = [\boldsymbol{a}_4, \boldsymbol{a}_5, \boldsymbol{a}_6], \\ &\boldsymbol{g}(\boldsymbol{z}) = [g_1(z_1), g_2(z_2), g_3(z_3)]^T, \quad \hat{\boldsymbol{g}}(\hat{\boldsymbol{z}}) = [g_4(z_4), g_5(z_5), g_6(z_6)]^T, \\ &z_\alpha = x_1 + p_\alpha x_2 \quad (\alpha = 1 - 6). \end{aligned} \quad (2.7)$$

As to the responses for the stresses, it is known that the stress components $\boldsymbol{t}_1 = [\sigma_{11}, \sigma_{12}, \sigma_{13}]^T$ and $\boldsymbol{t}_2 = [\sigma_{21}, \sigma_{22}, \sigma_{23}]^T$ are computed by the following formula (Ting, 1996)

$$\boldsymbol{t}_1 = -\boldsymbol{\varphi}_{,2} + \rho v^2 \boldsymbol{u}_{,1}, \quad \boldsymbol{t}_2 = \boldsymbol{\varphi}_{,1}, \quad (2.8)$$

where $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \varphi_3]^T$, the stress function vector, is defined as

$$\boldsymbol{\varphi} = \sum_{\alpha=1}^{6} \boldsymbol{b}_\alpha g_\alpha(Z_\alpha), \quad \text{or} \quad \boldsymbol{\varphi} = \boldsymbol{B}\boldsymbol{g}(\boldsymbol{z}) + \hat{\boldsymbol{B}}\hat{\boldsymbol{g}}(\hat{\boldsymbol{z}}), \quad (2.9)$$

where

$$\boldsymbol{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3], \quad \hat{\boldsymbol{B}} = [\boldsymbol{b}_4, \boldsymbol{b}_5, \boldsymbol{b}_6], \quad (2.10)$$

$$\begin{aligned} \boldsymbol{b}_\alpha &= \left(\boldsymbol{R}^T + p_\alpha \boldsymbol{T}\right)\boldsymbol{a}_\alpha \\ &= -p_\alpha^{-1}(\boldsymbol{Q} - \rho v^2 \boldsymbol{I} + p_\alpha \boldsymbol{R})\boldsymbol{a}_\alpha, \quad (\alpha = 1 - 6). \end{aligned} \quad (2.11)$$

When speed $v = 0$, the problem corresponds to elastostatics. There are three different problems needed to be classified for $v \neq 0$, i.e., subsonic, supersonic and transonic problems, as mentioned previously. For supersonic problem, there are further three cases, called S1, S2 and S3 cases, needed to be distinguished, depending on how many pairs of eigenvalues being real occurred in the supersonic problem. The three cases are classified as follows:

(a) *S1 case:* Only one pair of p_α being real (say, p_3 and p_6), and the other two pairs remaining complex conjugate (say $p_4 = \bar{p}_1$ and $p_5 = \bar{p}_2$).
(b) *S2 case:* Two pairs of p_α being real (say, the pair p_2, p_5 and the pair p_3, p_6), and only one pair remaining complex conjugate (say $p_4 = \bar{p}_1$).
(c) *S3 case:* All three pairs of p_α being real.

In the next section, the responses for these three supersonic cases will be presented analytically for general anisotropic materials. The subsonic responses for general anisotropic materials will also be addressed for the purpose of subsequent developments.

3. Supersonic responses for general anisotropic materials

The problem under consideration is shown in Fig. 1 where a point force $f = [f_1, f_2, f_3]^T$ moves steadily with speed v over the surface of a half-plane solid ($x_2 \geqslant 0$). The boundary condition on the surface is

$$\boldsymbol{t}_2(x_1) = \boldsymbol{\varphi}_{,1}(x_1) = -\boldsymbol{f}\delta(x_1), \quad (3.1)$$