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Microbuckling of a doublewalled carbon nanotube embedded in an elastic matrix

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ABSTRACT

Microbuckling of a doublewalled carbon nanotube (DWCNT) in an elastic (polymer) matrix is studied. The investigations are made within the scope of the piecewise homogeneous body model by utilizing the three-dimensional linearized theory of stability of deformable bodies. Flexural and axisymmetric microbuckling modes are considered. The DWCNT is modeled as concentrically-nested two circular hollow cylinders between which there is free space. It is assumed that on the inner surface of the outer tube (cylinder) and on the outer surface of the inner tube (cylinder) of the DWCNT full slipping conditions occur. At the same time, it is assumed that the difference between the radial displacements of the adjacent surfaces of the tubes resists with the van der Waals forces. On the interface between the matrix and DWCNT complete contact conditions are satisfied. Numerical results on the influence of the problem parameters on the critical deformation are presented and discussed. Also, numerical results related to the cases where the interlayer space is ignored and where full contact between the tubes is assumed are presented and compared with the mentioned results. In particular, it is established that full slipping between the tubes causes the values of the critical deformation to decrease significantly with respect to those obtained in the case where complete contact conditions occur between the tubes. Moreover, it is established that an increase in the values of the van der Waals forces also causes a decrease in the values of the critical compressing strain and the magnitude of this decrease depends on the thicknesses of the tubes of the DWCNT.

1. Introduction

During the last 20 years, after the discovery of carbon nanotubes (CNT) by Iijima (1991), a lot of investigations have focused on studying the mechanical behavior of single walled and multiwalled CNT (SWCNT and MWCNT) as structural elements (see Ru, 2000; Shen, 2004; Thai, 2012 and references listed therein), as well as on the behavior of the SWCNT and MWCNT embedded in polymer and metal matrices (see Jochum and Grandidier, 2004; Lourie et al., 1998; Ru, 2001; Murmu and Pradhan, 2009 and others which are listed in these references). Note that in these works the continuum approach, i.e., the continuum mechanics of deformable bodies was employed to study the corresponding problems. The applicability of continuum solid mechanics concepts to describing the behavior of the nano-objects was discussed in papers by Young et al. (2012), Guz (2012), Duan et al. (2009) Windle (2007) and etc. Study of the mechanical behaviors of the CNT using applicable areas of the continuum approach, and other types of nanostructures were discussed by Harik (2001), Guz and Rushchidsky (2003), Guz and Rushchidsky (2012) and others.

Here we consider a brief review of the investigations related to the stability loss of the CNT embedded within an elastic medium. Note that one of the prior studies in this field was made by Ru (2000), in which, by employing the Euler beam theory, column buckling of the MWCNT embedded in an elastic medium was studied. The Winkler type model is used to simulate the interaction of the MWCNT with the surrounding elastic medium. It is assumed that each of the nested concentric nanotubes is an individual hollow column, and that the deflections of the columns are coupled to each other through the van der Waals interaction between adjacent nanotubes.

In another paper by Ru (2001) an axially compressed buckling of the doublewalled CNT (DWCNT) embedded in an elastic medium was studied. A double-shell model within the scope of the Kirchhoff–Love theory was used to describe the buckling of the DWCNT and the response of the surrounding medium to this buckling was presented by the Winkler foundation model. Moreover, the interaction between the outer and inner tubes is modeled by the van der Waals forces. In a paper by Shen (2004), within the scope of the same assumptions and equations used in a paper by Ru (2001), buckling of the DWCNT under hydrostatic pressure was investigated.

In a paper by Murmu and Pradhan (2009), buckling analysis of a SWCNT embedded in an elastic medium was made based on the non-local elasticity theory by employing the Timoshenko beam

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theory. Both Winkler-type and Pasternak-type models are employed to simulate the response of the surrounding elastic medium to the buckling of the SWCNT.

Note that in the foregoing and other papers related to the buckling of the MWCNT embedded in an elastic medium, the MWCNT is modeled as a column or as a shell with a finite length. Values of the critical forces or critical strains determined in these works depend on the end conditions given for the corresponding columns or shells. Consequently, the results obtained and the problems in these works can be related to the buckling of the short CNT for which the ratio of the length to the diameter is not more than 4.5. But for the investigation of the buckling of the long CNT (for which the above-mentioned ratio is significantly more than 4.5) embedded in elastic matrix the more suitable model is the model consisting of the CNT with infinite length embedded in infinite elastic matrix. In the latter case, according to Biot (1965), Guz and Rushchidsky (2012), Akbarov (2013), Akbarov and Guz (2000) and others, the buckling of the CNT is called as a microbuckling and it is obtained that the values of the critical parameters depend mainly on the mechanical and geometrical relations of the constituents of the considered system, but do not depend of the length of the CNT. Therefore, in papers by Jochum and Grandidier (2004), Guz (2006) and others, for the study of the microbuckling of the carbon fibers or CNTs, they are modeled with columns (solid cylinders) or shells (hollow cylinders) with infinite length in the surrounding elastic medium. It should be noted that in the paper by Jochum and Grandidier (2004) microbuckling of a single carbon fiber in an epoxy matrix is examined experimentally but under mathematical modeling of this buckling, which is made by the use of the Euler beam theory, it is assumed that the fiber is embedded in an infinite circular cylinder of the matrix and the cross section of this cylinder has a finite radius. As a result of this assumption, the expressions obtained for the critical parameters must depend on this radius.

The microbuckling of the SWCNT in a polymer matrix was considered in depth in the aforementioned paper by Guz (2006) and this consideration was made by utilizing the Three-Dimensional Linearized Theory of Stability of Deformable Bodies (TDLTSDB). It should be noted that similar investigations, which have been made for internal stability loss in the structure of traditional fibrous composites, were also made in the last twenty years of the 20th century by Guz and his students. Systematic consideration of these investigations was made in the monographs by Guz (1990), Guz and Rushchidsky (2012) and Akbarov (2013).

However, up to now there has not been any investigation on the microbuckling of the MWCNT embedded in an elastic (polymer) matrix. Note that such investigations have important significance for estimation of the failure forces in compression of composites reinforced with MWCNTs. In addition, such investigations may be used to understand the mechanical behavior of the MWCNTs in an elastic matrix. Therefore, in the present paper an attempt is made for investigation of the microbuckling of the DWCNT in an elastic matrix. The investigation is carried out by using the TDLTSDB within the scope of the piecewise homogeneous body model. The DWCNT is modeled as concentrically-nested two circular hollow cylinders between which there is free space. It is assumed that on the inner surface of the outer tube (cylinder) and on the outer surface of the inner tube (cylinder) of the DWCNT full slipping conditions occur.

2. Formulation of the problem

We consider an infinite elastic matrix containing a DWCNT which is modeled as concentrically-nested two circular hollow cylinders with an infinite length between which there is free space (Fig. 1). It is assumed that on the inner surface of the outer tube

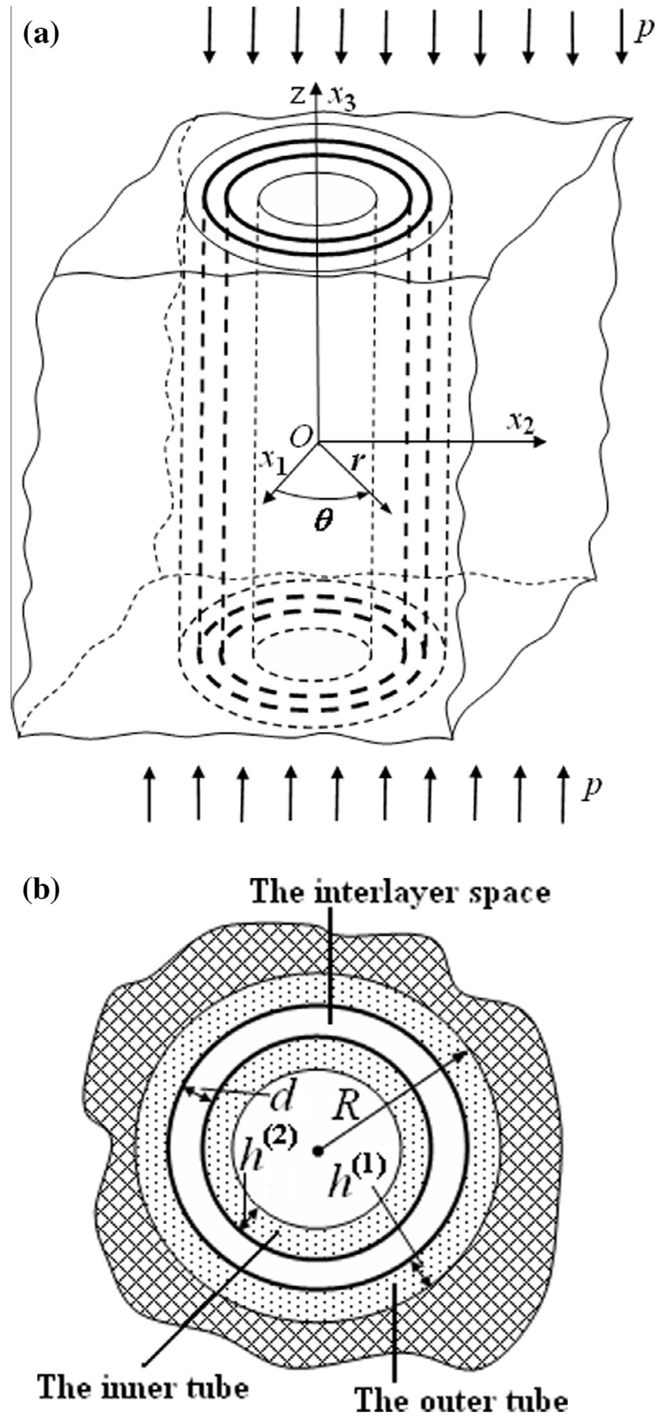

Fig. 1. The geometry of the DWCNT (a) and its cross section (b).

(cylinder) and on the outer surface of the inner tube (cylinder) of the DWCNT, full slipping conditions occur. At the same time, it is assumed that the difference between the radial displacements of the adjacent surfaces of the tubes resists with the van der Waals forces. We suppose that the external radius of the outer layer-cylinder is R; the thickness of the outer and inner layers of the cylinders we denote through $h^{(1)}$ and $h^{(2)}$, respectively (Fig. 1b), and the thickness of the interlayer spacing we denote by d. For the case under consideration, below we will use the subscriptions (1), (2) and (m) to denote the quantities related to the outer, inner cylinders and matrix materials respectively. Moreover, to denote the quantities related to the pre-critical state we will use the upper index 0. In the natural state, we associate the cylindrical $Or\theta z$ and Cartesian