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Crack tip field in a linear elastic–plastic strain-hardening material

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ABSTRACT

The strip necking model for strain-hardening materials is studied in this paper, in which the stress distributed over the strip necking zone is assumed to be ultimate stress. The bi-linear stress–strain relation which can model certain features of plastic flow is adopted in this model. The stress and strain fields are calculated based on this model in this paper. The size of the strip necking region is determined by balancing the stress intensity factor due to remote loading with that due to assumed closing forces equal to the ultimate tensile strength of the material distributed over the strip necking zone. It is interesting that the strip necking region size and the crack tip opening displacement depend not only on the remote load, but also the material hardening parameters, which is different from the results of strip yield model. The results agree with experiments well, and the model has wider application.

1. Introduction

It is well known that a plastic zone would appear near the crack tip in an elastic–plastic material under increasing load. It is true that the plastic zone size is small compared to the crack size in a material at low load levels. Dugdale (1960) investigated the plastic yielding near a crack tip in a thin metal sheet and proposed the classical strip yield model. After generalizing the essential ideas of Dugdale, the strip yield model was applied in other materials. Gao et al. (1997) proposed the strip electric saturation model in piezoelectric materials, and Zhao and Fan (2008) presented strip electric–magnetic breakdown model in a magnetoelectroelastic medium. A relation was obtained between the extent of plastic yielding and external loading applied. Based on the strip yield model, Burdekin and Stone (1966) studied the crack tip opening displacement and provided the basis for the design curve which is examined experimentally using mild steel specimens of vastly differing dimensions. When more experimental results were available, Burdekin and Dawes (1971) revised the design curve by raising the linear portion above the upper limit of the scatter band of results. Dawes (1974) modified the toe region of the design curve to increase its safety, and it is also simpler to use.

Strain-hardening plays an important role of producing unique stress and strain fields in the plastic zone near a stationary crack tip. Hutchinson (1968) and Rice and Rosengren (1968) proposed the well-known HRR singular fields near the crack tip in a power law hardening material. The strength of the singularity is uniquely determined by *J*-integral defined by Rice (1968). Since that time, much work has been done in the area of the elastic–plastic fracture mechanics. Amazigo and Hutchinson (1977) investigated singular stress and strain fields at the tip of a crack growing steadily and quasi-statically in an elastic–plastic strain-hardening material characterized by J_2 flow theory and a bi-linear effective stress–strain curve. Using the HRR theoretical developments as a foundation, Xia et al. (1993) carried out a higher-order asymptotic analysis of a stationary crack in a power law hardening material for plane strain, Mode I. In addition, Mode II plane strain crack was also studied by Xia and Wang (1992). Two parameter approaches as the more effective elastic–plastic fracture criterion is developed. Later Wei and Wang (1995) presented a modified two parameter criterion based on the asymptotic solution of five terms.

Larsson and Carlsson (1973) studied the influence of non-singular stress terms and specimen geometry on small scale yielding at crack tip in elastic–plastic materials with finite element method. Further implications of the non-singular stress term for crack tip deformations and fracturing is examined by Rice (1974). It is suggested that its effect on the crack tip parameters, such as the opening displacement and *J*-integral, is less pronounced than its effect on the yield zone size. Tvergaard and Hutchinson (1992) studied the relation between crack growth resistance and fracture process parameters in elastic–plastic solids with an idealized traction–separation law. Later, Tvergaard and Hutchinson (1994) considered the effect of the non-singular *T*-stress (Williams, 1957) on Mode I crack growth resistance in a ductile solids.

If a tensile stress is applied to a thin crack plate, a strip necking region was observed ahead of a crack tip in the experiments by Schaeffer et al. (1971). The strip necking zone ahead of a crack tip is studied theoretically in this paper. The stress in the strip necking zone should not be greater than ultimate stress which is

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Nomenclature

E, $E_{\tan}$	the slope of the piecewise-linear stress–strain curve
v	Possion's ratio
$\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$	the Kronecker delta
σ_{ij}	stress
ε_{ij}	strain
$\bar{\sigma}_{ij}$	non-dimensional stress
$\bar{s}_{ij}$	non-dimensional stress deviator
$\bar{\sigma}_e$	non-dimensional effective stress
σ_∞	the remote load
σ_Y	yield stress
ε_Y	yield strain
σ_u	ultimate stress
K	stress intensity factor
U	stress function
$\phi(z),\ \chi(z),\ \omega(z),\ \Omega(z)$	analytic functions for displacement
$\Phi(z),\ \Psi(z)$	analytic functions for stress and strain
$u_x,\ u_y$	displacement in x and y direction
a	half-length of the real crack
c	half-length of the effective crack
r_n	the strip necking region size
r_s	the strip yielding region size
δ	crack tip opening displacement
Φ	non-dimensional crack tip opening displacement

a material constant determined by the uniaxial tensile test. The ultimate stress is the maximum tensile force divided by the initial transverse section area of the specimen. Therefore, the stress in y direction is assumed to be ultimate stress distributed over the strip necking zone before the crack propagation in this model. Linear strain-hardening is considered and the stress–strain relation is reduced to be a linear mechanical problem when the plastic zone is very small relative to the crack length. The size of the strip necking region and the crack opening displacement are obtained based on the strip necking model for bi-linear strain-hardening materials in this paper, and it agrees with experimental results well.

2. Basic equations

In order to analyze the problem conveniently, the stress σ_{ij} and strain s_{ij} is non-dimensionalized by a yield stress σ_Y and the corresponding yield strain $\varepsilon_Y = \sigma_Y/E$ respectively,

$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\sigma_Y}, \qquad \bar{s}_{ij} = \frac{s_{ij}}{\varepsilon_Y}. \tag{1}$$

The first invariant of the stress deviation $\bar{s}_{ij}$ and the effective stress $\bar{\sigma}_e$ are defined respectively by

$$\bar{s}_{ij} = \bar{\sigma}_{ij} - \frac{1}{3}\bar{\sigma}_{kk}\delta_{ij}, \tag{2}$$

$$\bar{\sigma}_e^2 = \frac{3}{2}\bar{s}_{ij}\bar{s}_{ij}. \tag{3}$$

In a simple tensile test, there exists plastic deformation in the materials when the stress is larger than the yield stress. The stress–strain relation in a linear strain-hardening material is shown in Fig. 1. This bi-linear approximation does model certain features of plastic flow. Then the bi-linear stress–strain relation can be formulated as follows (Hutchinson, 1968).

$$\bar{\varepsilon}_{ij} = (1+v)\bar{\sigma}_{ij} - v\bar{\sigma}_{pp}\delta_{ij} + \lambda(1-\bar{\sigma}_e^{-1})\bar{s}_{ij}, \tag{4}$$

where v is Possion's ratio and λ is a parameter determined by the following expressions,

$$\lambda = \tfrac{3}{2}\left(\tfrac{E}{E_{\tan}} - 1\right), \quad \bar{\sigma}_e > 1; \qquad \lambda = 0, \quad \bar{\sigma}_e < 1. \tag{5}$$

Based on the result obtained by Hutchinson (1968), the effective stress is

$$\bar{\sigma}_e = K\bar{r}^{-\frac{1}{2}}\left(\cos^2\frac{1}{2}\theta + \frac{3}{4}\sin^2\theta\right)^{\frac{1}{2}}, \tag{6}$$

where $\bar{r} = r/a$, and a is the half length of the crack. For applied stress is sufficiently low, the plastic zone is very small relative to the crack length, i.e. $\bar{r} \ll 1$. Therefore, $\bar{\sigma}_e^{-1}$ is very small and can be neglected for calculating the dominant singularity in the necking zone near the crack tip. Then Eq. (4) can be reduced into

$$\bar{\varepsilon}_{ij} = (1+v)\bar{\sigma}_{ij} - v\bar{\sigma}_{pp}\delta_{ij} + \lambda\bar{s}_{ij}. \tag{7}$$

Unless otherwise stated, λ is $3(E/E_{\tan} - 1)/2$ in the rest of the paper. Substituting Eq. (1) into Eq. (7) yields

$$\varepsilon_{ij} = (1+v+\lambda)\frac{\sigma_{ij}}{E} - \left(v + \frac{\lambda}{3}\right)\frac{\sigma_{pp}}{E}\delta_{ij}. \tag{8}$$

The dominant singularity can be derived based on the constitutive equation (8) in the plastic zone (Hutchinson, 1968). Obviously, the elastic zone and the plastic zone have different constitutive equations. In fact, we can get the constitutive equation in the elastic zone, which is outside the plastic zone when λ is zero,

$$\varepsilon_{ij} = (1+v)\frac{\sigma_{ij}}{E} - v\frac{\sigma_{pp}}{E}\delta_{ij}. \tag{9}$$

The stress can be obtained from a stress function by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2}, \qquad \sigma_{yy} = \frac{\partial^2 U}{\partial x^2}, \qquad \sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y}. \tag{10}$$

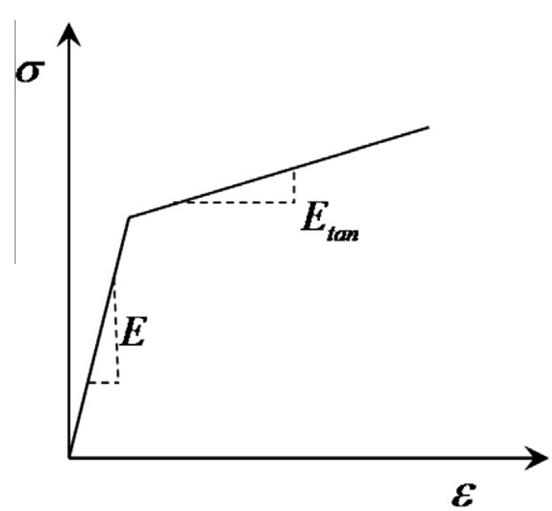

Fig. 1. Bi-linear stress–strain relation.