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Constitutive modeling of pressure dependent plasticity and fracture in solder joints

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ABSTRACT

A new constitutive model describing the pressure dependence of plasticity and fracture in solder joints is proposed. The pressure dependence of the solders flow stress is obtained from the Peierls stress necessary to move mobile dislocations. Conservation of volume during plastic deformation is achieved through a deviatoric associated flow rule. Fracture is considered as a result of damage accumulation depending on the amount of plastic strain and triaxiality of stress the sample has experienced. The constitutive model was implemented in the FEM code Abaqus using the user subroutine UMAT. The model predictions were compared to experimental results of tensile tests performed with Sn-3.5Ag solder joints of various thicknesses. Ultimate tensile strength and ductility showed a strong dependency on the solder joints geometry. It is demonstrated that the model can nicely be fitted to experimental results.

1. Introduction

Due to their applications in electronic industry, solder joints represent the critical links in determining the lifetime of electronic parts. For several decades, lead containing solders were widely used as solder material, because of their low melting temperature and excellent mechanical behavior. However, due to concerns regarding environment and health, they are now eliminated and substituted by lead-free solder alloys. This has lead to renewed interest in constitutive models describing the elasto-plastic properties of solders. Several investigations in recent years have shown that lead-containing and lead-free solders may be treated within the same theoretical framework using different material parameters for different alloys: The strain rate-dependent creep properties are usually modeled on the basis of the Anand model (Anand, 1985; Amagai et al., 2002) the temperature and strain rate dependence of which has been further improved by several authors (Chen et al., 2004; Pei and Qu, 2005; Bai et al., 2009). On the other hand, the time independent behavior due to mechanical shock or vibration loading can be modeled using an approach of kinematic hardening (Wiese and Rzepka, 2004). A clear analogy of material models for lead-free and lead-containing solder alloys was found although Sn-Ag and Sn-Ag-Cu solders show higher strength and less ductility compared to the eutectic Sn-37Pb solder alloy.

It is well known that the strength of solder alloys is affected by microstructure and size effects (Khatibi et al., 2012). Aside from the mechanical properties of the solder itself the strength of a solder joint also depends on its geometry and sometimes even on the properties of the base material. Because of the boundary condition at the interface to the stronger base material, the cross contraction inside the solder is obstructed during tensile deformation. In consequence, the solder joint can carry a tensile load which is much higher than the tensile strength of the solder material alone (Zimprich et al., 2008; Hedge et al., 2009). This effect was investigated with finite element analysis compared to tensile tests of Sn-4.0Ag-0.5Cu (Cugnoni et al., 2006) and Sn-3.5Ag (Lederer et al., 2012) solder joints. In spite of the fact that very similar results were found in experiment and simulation, the theoretical analysis seems to overestimate the increase of joint strength compared to the properties of the pure solder material. In the present study, this is attributed to a pressure dependence of the solders flow stress and fracture behavior. Therefore, a new pressure dependent model for solder materials is developed which is validated for solder joints of various thicknesses exposed to tensile stress. In the case of thin joints, the stress triaxiality inside the solder reaches a remarkable high value. Thus, the required material parameters can be extracted from a numerical fit of the model to experimental data.

An effect of superimposed hydrostatic pressure on the deformation and fracture behavior of metals is generally accepted (Bridgman, 1952). Extensive experimental studies have shown that an increase of hydrostatic pressure leads to an increase of flow stress and ductility (Lewandowski and Lowhaphandu, 1998). However, the theoretical modeling of pressure dependent plasticity has revealed a fundamental problem: Early theories based on the normality flow rule predicted a considerable increase of volume during plastic deformation. While this approach (Drucker and Prager, 1952) was successfully applied to granular materials, it is in general inadequate to explain the behavior of metals (Spitzig and Richmond,

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1984). In fact, there is no need to associate the pressure dependence of yielding in metals with irreversible plastic dilatancy. Hence, pressure dependent yielding in metals is better described by a deviatoric associated flow rule (Bai and Wierzbicki, 2008) postulating conservation of volume during plastic deformation.

Several pressure dependent fracture models are known in literature stating that a triaxiality of tensile stress reduces the amount of plastic strain to fracture. This behavior may be derived from the assumption of porosity due to cylindrical (McClintock, 1968) or spherical holes (Rice and Tracey, 1969) in a material subjected to plastic flow. But it has to be questioned whether this model may be applied to solder joints exhibiting high stress triaxiality and at the same time enhanced tensile strength compared to bulk solder material. Nevertheless, a similar description of plastic strain to fracture may be obtained from a model of damage accumulation (Johnson and Cook, 1985). Therefore, a fracture criterion resulting from damage accumulation will be proposed which fully agrees with the flow rule of the present approach.

2. Formulation of flow rule and fracture criterion

2.1. Pressure dependence of the flow stress

Plastic flow in metals is mainly due to glide of crystal dislocations, and the stress required to move a mobile dislocation is called the Peierls stress. While propagating through the crystal lattice a dislocation must override a periodic potential. For an edge dislocation in a cubic lattice a shear stress of

$$\tau = c_1 \cdot \exp\left\{-\frac{c_2 \cdot d}{b}\right\} \tag{1}$$

is needed to shift atoms at the dislocation core from one stable lattice position over the barrier to the next stable position (Peierls, 1940). Here, c_1 and c_2 are material parameters, d is the interplanar distance between neighboring lattice planes, and b is the Burgers vector of the dislocation. It has already been stated that the result of Eq. (1) may be further improved by numerical computations which are either based on quantum mechanical methods (Mainprice et al., 2008) or on an elastic variational principle of higher dimensionality (Schoeck, 2005, 2006). Nevertheless, we use the original Peierls model as starting point of our investigations, since we wish to obtain a closed formula for the pressure dependence of Peierls stress:

The hydrostatic pressure p is related to the stress tensor σ_{ij} by

$$p = -\sigma_m = -\frac{1}{3}Tr(\sigma_{ij}). \tag{2}$$

The pressure introduces a volume change

$$\varepsilon_{vol} = -\frac{p}{K} \tag{3}$$

of the material, where K is the bulk modulus and $\varepsilon_{vol} = Tr(\varepsilon_{ij})$ is the trace of the strain tensor. Thereby, the interplanar distance d occurring in Eq. (1) is altered. In the isotropic case, one obtains

$$\varepsilon_{vol} = 3 \cdot \frac{\Delta d}{d_0}, \tag{4}$$

where d_0 is the interplanar distance at zero pressure. Inserting the altered value for d into Eq. (1) yields

$$\tau = c_1 \cdot \exp\left\{-\frac{c_2 \cdot \left(d_0 - \frac{d_0 \cdot p}{3 \cdot K}\right)}{b}\right\}. \tag{5}$$

The critical shear stress at zero pressure is hereafter denoted as

$$\tau_0 = c_1 \cdot \exp\left\{-\frac{c_2 \cdot d_0}{b}\right\} \tag{6}$$

Therefore, Eq. (5) may now be written as

$$\tau = \tau_0 \cdot \exp\left\{\frac{c_2 \cdot d_0 \cdot p}{3 \cdot b \cdot K}\right\}. \tag{7}$$

Strictly speaking, a change in volume would also change the Burgers vector b of the crystal. However, we here argue that the width of the potential barrier to be overcome by the dislocation core rather depends on the atomic radius than on the Burgers vector. Therefore, we replace the Burgers vector b in Eq. (7) by $a \cdot c_3$, where a is the atomic radius and the constant c_3 is a geometry factor. Thus, Eq. (7) rewrites as

$$\tau = \tau_0 \cdot \exp\left\{\frac{c_2 \cdot d_0 \cdot p}{3 \cdot a \cdot c_3 \cdot K}\right\} \tag{8}$$

Next, the constants in the argument of exponential function are replaced by a single constant

$$\alpha = \frac{c_2 \cdot d_0}{3 \cdot a \cdot c_3 \cdot K}. \tag{9}$$

Consequently, the dependence of Peierls stress on the hydrostatic pressure now takes the form

$$\tau = \tau_0 \cdot \exp\{\alpha \cdot p\}. \tag{10}$$

Furthermore, one has to consider different contributions to the flow stress of a material. In fact, the Peierls stress represents only the least stress required to move a dislocation. During ongoing plastic deformation, dislocations are stopped at various obstacles which can be bypassed at a higher stress level. But effects of this kind are expected to show more or less the same pressure characteristics as described by Eq. (10). When the crystal lattice is stretched due to hydrostatic tension, the interplanar distance increases and dislocations have more space to circumvent obstacles. Hence, we claim that the pressure dependence of flow stress may be described by

$$\sigma_f = \sigma_0 \cdot \exp\{\alpha \cdot p\}, \tag{11}$$

whereby the flow stress σ_0 at zero pressure depends on the dislocation density and hence on the amount of plastic strain accumulated in the sample. The proportionality of critical shear stress and flow stress at a given pressure is here based on an analogy to the von Mises yield criterion (von Mises, 1913), which has already been justified on the basis of an octahedral shear stress yield criterion (Dowling, 1999). Eq. (11) obviously deviates from several other theories assuming a linear dependence of flow stress on hydrostatic pressure. The value of the material parameter α is, however, rather small and therefore Eq. (11) leads only to a moderate nonlinearity within the pressure range covered by the present investigation.

2.2. Strain hardening

In order to evaluate strain hardening, we express σ_0 as function of the plastic equivalent strain

$$\varepsilon^{pl}_{eqv} = \sqrt{\frac{2}{3}\varepsilon^{pl}_{ij} \cdot \varepsilon^{pl}_{ij}} \tag{12}$$

which plays the role of an internal state variable. Further, we use the general form of a hardening law which was successfully applied to lead-free Sn-4.0Ag-0.5Cu (Cugnoni et al., 2006):

$$\sigma_0\left(\varepsilon^{pl}_{eqv}\right) = \sigma_Y + Q\left(1 - \exp\left\{-B \cdot \varepsilon^{pl}_{eqv}\right\}\right) + K \cdot \varepsilon^{pl}_{eqv} \tag{13}$$

Here, σ_Y is the yield stress at zero plastic strain while Q, B and K are additional material parameters to be determined from numerical fitting to experimental data.