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Symmetry breaking in an initially curved micro beam loaded by a distributed electrostatic force

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ABSTRACT

The asymmetric buckling of a shallow initially curved stress-free micro beam subjected to distributed nonlinear deflection-dependent electrostatic force is studied. In order to highlight the symmetry breaking phenomenon and the approach to its analysis, the subsidiary simplified problem of a curved beam attached to a linearly elastic foundation, and subjected to uniformly distributed “mechanical” load, which is independent of deflections, is addressed first. The analysis is based on a two degrees of freedom reduced order (RO) model resulting from the Galerkin decomposition with linear undamped eigenmodes of a straight beam used as the base functions. Simple approximate expressions are derived defining the geometric parameters of beams for which an asymmetric response bifurcates from the symmetric one. The necessary criterion establishes the conditions for the appearance of bifurcation points on the unstable branch of the symmetric limit point buckling curve; the sufficient criterion assures a realistic asymmetric buckling bifurcating from the stable branches of the curve. It is shown that while the symmetry breaking conditions are affected by the nonlinearity of the electrostatic force, its influence is less pronounced than in the case of the symmetric snap-through criterion. A comparison between the RO model results and those obtained by direct numerical analysis shows good agreement between the two and indicates that the obtained criteria can be used to predict non-symmetric buckling in electrostatically actuated bistable micro beams.

1. Introduction

Initially curved beams (arches) loaded by concentrated or distributed transverse forces may exhibit bistability, namely, the existence of two different stable equilibria under the same loading. The transition between two stable states in these structures is commonly referred to as a snap-through buckling. The behavior of beams liable to the snap-through buckling is well understood and it is an established topic in structural mechanics (e.g., see Bažant and Cedolin, 1991; Dym, 1974; Seyranian and Elishakoff, 1989; Thompson and Hunt, 1973; Simitses, 1989; Timoshenko, 1961 and references therein. See Chen and Chang (2007), Mallon et al. (2006), Plaut (2009) and Plaut and Virgin (2009) for some of the recent contributions). On the most basic level, initially straight or slightly curved beams do not buckle under a transverse force, whereas sufficiently curved beams manifest a symmetric (limit point) snap-through and are bistable in the interval of the force between the snap-back (release) and snap-through values. When the initial curvature of the beam is higher than a certain value, the buckling is accompanied by a symmetry breaking and the appearance of non-symmetric buckling configurations.

Recently, a renewed interest in the mechanics of bistable beams stimulated additional studies (Das and Batra, 2009a,b; Krylov et al., 2008, 2011; Ouakad and Younis, 2010; Park and Hah, 2008; Pane and Asano, 2008; Saif, 2000; Qiu et al., 2004; Zhang et al., 2007) that were motivated by the progress in fabrication technologies and by the emerging of new applications in the realm of micro and nanoelectromechanical systems (MEMS and NEMS). The reason for this interest is twofold. On the one hand, micro and nano devices incorporating bistable structural elements have clear functional advantages in applications such as switches (Intaraprasonk and Fan, 2011), sensors (Southworth et al., 2010) and non-volatile memories (Charlot et al., 2008). On the other hand, over the past decade, electrostatically actuated initially straight double-clamped micro beam became a kind of benchmark problem, which was intensively used for the evaluation of various analytical, numerical and experimental approaches (see reviews Batra et al., 2007b; Nayfeh et al., 2003; Rhoads et al., 2008 and references therein). One of the distinguishing features of such a micro device is that it is loaded by an electrostatic force, which is a nonlinear function of the beam's deflections. For this reason, while being a relatively simple structure, a micro beam exhibits rich behavior and

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represents a convenient platform for analytical, numerical and experimental investigation of the abundant nonlinear phenomena at the microscale, which are rarely encountered or are difficult to envisage within large scale structures. As an example of this kind of phenomena, one can mention electrostatic (so-called pull-in) instability taking place in micro beams and associated with the softening nonlinearity of the electrostatic forces, which reduces the effective stiffness of the structure.[1]

In contrast to straight beams, initially curved electrostatically actuated double-clamped beams combine both geometric mechanical nonlinearity typical for bistable structures and generic electrostatic softening nonlinearity. As was recently shown by Das and Batra (2009a), Das and Batra (2009b), Krylov et al. (2008) and Zhang et al. (2007), these structures may exhibit sequential snap-through buckling and pull-in instability. Note that the limit point snap-through and the symmetry breaking criteria for beams subjected to a "mechanical" deflection-independent loading are fully dictated by the geometry of the beam itself-namely by the ratio between the initial elevation/curvature of the beam and its thickness – and is independent on the loading (Dym, 1974; Simitses, 1989). However, in the case of the electrostatic actuation, the snap-through behavior is affected by the nonlinearity of the electrostatic force parameterized by the initial distance between the beam and the electrode, as reflected in the symmetric (limit-point) snap-through criterion first obtained in Krylov et al. (2008) for an initially stress-free bell-shaped beam. It was shown, that in the case of the electrostatic loading, the snap-through may take place in beams with lower initial elevation/curvature when compared to the case of "mechanical" deflection-independent loading.

It should be noted, that while relatively a large body of work was devoted to the stability and dynamics of straight and curved micro beams, the number of studies dealing with the non-symmetric buckling of these structures loaded by configuration-dependent electrostatic forces is limited. Non-symmetric buckling of curved bell-shaped beams subjected to distributed electrostatic loading was illustrated in Krylov et al. (2011) by means of the reduced-order and computational models. Symmetry breaking in a similar structure was analyzed in more details in Das and Batra (2009b) and was found to have significant influence on the stability boundaries of the beam: the snap-through voltage was shown to be lower than predicted by the symmetric model. Initially straight micro beams buckled due to pre-stress and then actuated by electrostatic force engendered by the fringing fields were analyzed numerically in Krylov et al. (2011) and were shown to exhibit non-symmetric buckling for sufficiently high values of the initial pre-stress and consequently curvature. Non-symmetric pull-in configurations of initially flat annular membranes were considered in Batra et al. (2007a) and Pelesko et al. (2003), non-symmetric pull-in configurations of annular plates under electrostatic and Casimir forces were obtained numerically in Batra et al. (2006). However, no symmetry-breaking criteria were obtained in all these works.

In this work, we extend the stability analysis of electrostatically actuated initially curved stress-free micro beams to the non-symmetric configurations. Our goal is to highlight the leading phenomena taking place in this type of structure, to investigate the influence of the device parameters on its stability and to establish criteria of symmetry breaking. We develop simple approximate relations between the geometrical parameters of the structure (thickness, initial elevation and distance between the beam and the electrode), which should be satisfied in order to obtain the non-symmetric buckling and find the values of the beam's deflection and of the actuation voltage corresponding to the critical points. These criteria are in a sense an extension of the well-documented results obtained for mechanically loaded curved beams (see Dym, 1974 and Simitses, 1989) to the case of the intrinsic nonlinear electrostatic loading.

In the next section, the problem of an initially curved bell-shaped stress-free beam under a distributed electrostatic force is formulated, followed by the development of a reduced order (RO) model based on the Galerkin decomposition and limited to incorporate two-symmetric and non-symmetric-terms. Next, an auxiliary problem of a curved beam resting on a linear (i.e, constant, deflection-independent, stiffness) elastic foundation and loaded by a "mechanical" force, which is independent on deflections, is considered and snap-through and symmetry breaking criteria are obtained. Due to its simplicity, this problem represents a convenient framework allowing simple closed form expressions. Then, the two-mode RO model of the electrostatically loaded beam is considered and the stability criteria are obtained along with the critical values of the deflections and actuation voltages. In the last section, the approximate results are verified using the numerical solution of the governing equations of the beam. Main findings of the work are summarized in the conclusions.

2. Formulation

We consider a flexible initially curved double clamped prismatic micro beam of length L having a rectangular cross-section of width $\hat{b}$ and thickness $\hat{d}$ as shown in Fig. 1. The beam is made of homogeneous isotropic linearly elastic material with Young's modulus E. Since the width $\hat{b}$ of a microbeam is typically larger than it's thickness $\hat{d}$, an effective (plain strain) modulus of elasticity $\widetilde{E} = E/(1-\nu^2)$ is used, where ν is Poisson's ratio. The initial shape of the beam is described by the function $\hat{w}_0(\hat{x}) = \hat{h}z_0(\hat{x})$, where $\hat{h}$ is the initial elevation of the beam's central point above it's ends, and $z_0(\hat{x})$ is a non dimensional function such that $\max_{\hat{x}\in[0,L]}[z_0(\hat{x})] = 1$. The beam is subjected to a distributed electrostatic force provided by an electrode located at a distance $\hat{g}_0$ (the gap) from the beam and extended beyond it's ends.

We assume that $\hat{d} \ll L$, $\hat{h} \ll L$ and that the deflections, while comparable with the thickness of the beam, are small with respect to the beam's length. Under these assumptions, the beam's behavior is described in the framework of the Euler–Bernoulli theory combined with the shallow arch approximation and is governed by the following equilibrium equations (Simitses and Hodges, 2006; Villagio, 1997).

$$\widetilde{E}A\frac{\partial}{\partial \hat{x}}\left(\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{1}{2}\left(\frac{\partial \hat{w}}{\partial \hat{x}}\right)^2 - \frac{1}{2}\left(\frac{\partial \hat{w}_0}{\partial \hat{x}}\right)^2\right) = 0 \quad (1)$$

$$\widetilde{E}I_{yy}\left(\frac{\partial^4 \hat{w}}{\partial \hat{x}^4} - \frac{\partial^4 \hat{w}_0}{\partial \hat{x}^4}\right) - \widetilde{E}A\left(\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{1}{2}\left(\frac{\partial \hat{w}}{\partial \hat{x}}\right)^2 - \frac{1}{2}\left(\frac{\partial \hat{w}_0}{\partial \hat{x}}\right)^2\right)\frac{\partial^2 \hat{w}}{\partial \hat{x}^2} = \hat{f}^e \quad (2)$$

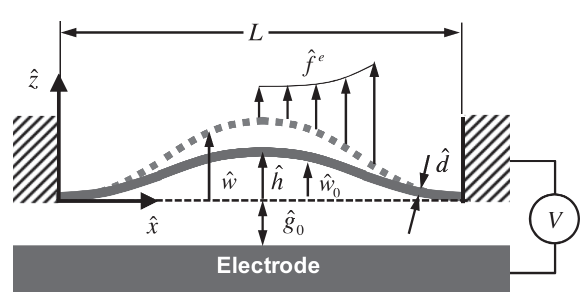

Fig. 1. Model of an initially curved double-clamped beam actuated by distributed electrostatic force. The dashed line corresponds to the deformed configuration. Positive directions of the beam's deflection and of the loading are shown.

[1] In large scale structures, the voltage required to initiate this instability is much larger than the electric breakdown voltage, the influence of the electrostatic force on the structure's deflection is not pronounced and the mechanical end electrostatic problems are decoupled.