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Perturbation analysis of Mode III interfacial cracks advancing in a dilute heterogeneous material

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ABSTRACT

The paper addresses the problem of a Mode III interfacial crack advancing quasi-statically in a heterogeneous composite material, that is a two-phase material containing elastic inclusions, both soft and stiff, and defects, such as microcracks, rigid line inclusions and voids. It is assumed that the bonding between dissimilar elastic materials is weak so that the interface is a preferential path for the crack. The perturbation analysis is made possible by means of the fundamental solutions (symmetric and skew-symmetric weight functions) derived in Piccolroaz et al. (2009). We derive the dipole matrices of the defects in question and use the corresponding dipole fields to evaluate "effective" tractions along the crack faces and interface to describe the interaction between the main interfacial crack and the defects. For a stable propagation of the crack, the perturbation of the stress intensity factor induced by the defects is then balanced by the elongation of the crack along the interface, thus giving an explicit asymptotic formula for the calculation of the crack advance. The method is general and applicable to interfacial cracks with general distributed loading on the crack faces, taking into account possible asymmetry in the boundary conditions.

The analytical results are used to analyse the shielding and amplification effects of various types of defects in different configurations. Numerical computations based on the explicit analytical formulae allows for the analysis of crack propagation and arrest.

1. Introduction

Analytic solutions for a crack propagating in a *homogeneous* elastic solid containing a finite number of small defects (elastic and rigid inclusions, microcracks, voids) have been derived in Bigoni et al. (1998), Valentini et al. (1999), and Movchan et al. (2002) on the basis of the dipole matrix and weight function approach. The dipole field describes the leading-order perturbation produced by a small defect placed in a smooth stress field and gives rise to "effective" tractions applied along the crack faces, that is, ideal tractions which produce the same perturbation as that of the defect in question. Furthermore, the weight functions allow for the derivation of the corresponding perturbation of the stress intensity factor as weighted integral of the "effective" tractions. The method is general and applicable to both 2D and 3D cases and to defects of different type and shape, provided that the corresponding dipole matrix is appropriately constructed and the weight functions for the corresponding unperturbed cracked body are available.

Problems of a macrocrack interacting with microcracks have been analysed by Romalis and Tamuzh (1984) under the influence of mechanical loading and by Tamuzs et al. (1993) under the influence of heat flux, using the analytic functions and singular integrals approach (Muskhelishvili, 2008). The possible closure of crack surfaces and the consequent appearance of a contact zone have been considered in Tamuzs et al. (1994, 1996). An elastic problem involving collinear configuration of microcracks ahead of a macrocrack was solved independently by Rubinstein (1985) and Rose (1986). Asymptotic models of a semi-infinite crack interacting with microcracks have been developed by Gong and Horii (1989) and Meguid et al. (1991) using complex potentials and the superposition principle. Jin et al. (2007) considered the interaction between a Mode III interface crack and a screw dislocation dipole. A review of publications on macro–microdefect interaction problem is given in Tamuzs and Petrova (2002).

Recently, Piccolroaz et al. (2009) derived the symmetric and skew-symmetric weight functions for a semi-infinite two-dimensional interfacial crack, thus disclosing the possibility of applying the dipole matrix approach to the propagation of cracks along the interface in heterogeneous materials with small defects. The weight functions constructed in Piccolroaz et al. (2009) are of the generalised type and thus applicable to any type of boundary

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conditions along the crack faces. The singular perturbation associated with a small crack advance was also obtained there. It is worth noting that the availability of the skew-symmetric function for the problem under consideration is essential since the "effective" loading produced by the defects on the crack faces is not symmetrical, in general. In the present paper, we analyse the scalar case of antiplane shear loading. The full vector problem will be addressed elsewhere.

The paper is organised as follows. The formulation of the problem is outlined in Section 2, which includes also preliminary results on the unperturbed problem. Section 3 is devoted to the perturbation analysis, in particular the derivation of the dipole fields for several types of defects and the analysis of singular perturbation associated with the crack advance. Section 4 provides a number of numerical results based on the explicit analytical formulae. In particular, shielding and amplification effects of different defect configurations on the crack-tip field are presented. Simplified asymptotic formulae for the limiting case of loading applied at large distance from the crack tip are also derived in this section (confirming and extending the results presented in Mishuris et al. (in press), where only linear defects were considered). The possibility to design a neutral configuration for any given force system distributed along the crack faces is discussed. Finally, the crack propagation and arrest produced by the defects under consideration is analysed. In the appendix, we derive the dipole matrix for an elliptic elastic inclusion placed in a homogeneous antiplane field.

2. Problem formulation and preliminary results

We consider a two-dimensional composite structure consisting of a bimaterial matrix (two dissimilar elastic half-planes $\Omega_{\pm}$) containing a dilute distribution of inclusions, microcracks and rigid line inclusions, see Fig. 1. The two materials constituting the matrix are assumed to be linear elastic and isotropic, with shear moduli denoted by μ_+ and μ_-, respectively. All interfaces between different phases are assumed to be perfect, that is, the displacements and tractions remain continuous across the interface.

We introduce the following notations. Let $g_\varepsilon \subset \Omega_+$ be a small elastic inclusion of diameter $2\varepsilon l_1$ centred at the point $\boldsymbol{Y}_1 = (a_1, h_1)$. The shear modulus of the inclusion is denoted by μ_i, and it can be greater or smaller than the shear modulus μ_+ of the surrounding material, so that both stiff and soft inclusions are considered. The notation $\gamma_2^\varepsilon \subset \Omega_-$ is used for a microcrack of the length $2\varepsilon l_2$, centred at the point $Y_2 = (a_2, h_2)$ and making an angle $\alpha_2 > 0$ with the positive direction of the x_1-axis. By $\gamma_3^\varepsilon \subset \Omega_+$ we denote a movable rigid line inclusion of the length $2\varepsilon l_3$, centred at the point $Y_3 = (a_3, h_3)$ and making an angle $\alpha_3 > 0$ with the positive direction of the x_1-axis. Although these notations refer specifically to Fig. 1, the formulation can be easily extended to problems with different number and type of defects, as those considered in Section 4.

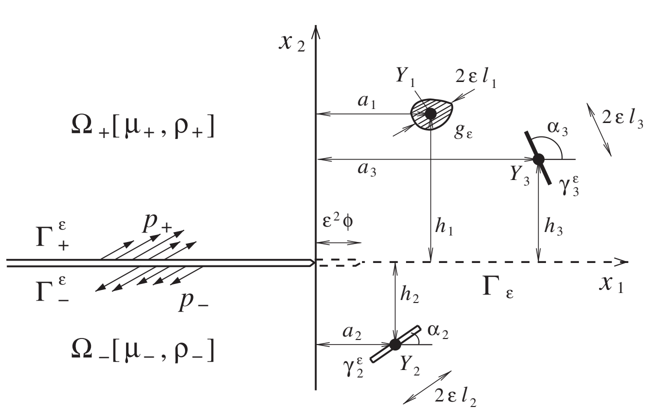

Fig. 1. Geometry of the problem: interface crack in a bimaterial plane with defects: g_ε denotes a small elastic inclusion, γ_2^ε a microcrack, and γ_3^ε a rigid line inclusion; $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, $\boldsymbol{Y}_3$ are the "centres" of the defects.

We assume that a semi-infinite interface crack M_ε advances quasi-statically along the interface Γ_ε connecting the half-planes, and we denote the uniform advance of the crack by $\varepsilon^2\phi$. Here and in the sequel, $\varepsilon > 0$ is a small dimensionless parameter. The reason for the order ε^2 in the crack advance will be clear in Section 3, where we perform the asymptotic analysis.

We assume that the composite is dilute, that is, the small defects are distant from each other so that the interaction between them can be neglected. Consequently, we can model the three cases of an elastic inclusion, of a microcrack and of a rigid line inclusion separately. It is possible then to extend the results to a finite number of defects by superposition using the linearity of the problem, provided that the distance between defects remains finite.

An external loading $p_\pm$ is applied to the crack faces $\Gamma_\pm^\varepsilon$ and it is assumed to be self-balanced such that the principal force vector is zero, that is,

$$\int_{\Gamma_+^\varepsilon} p_+ d\boldsymbol{x} - \int_{\Gamma_-^\varepsilon} p_- d\boldsymbol{x} = 0. \tag{1}$$

We assume that the loading $p_\pm$ on the crack faces vanishes in a neighbourhood of the crack tip.

The problem is then formulated in terms of the Laplace equation

$$\Delta u_\pm(x_1, x_2) = 0, \quad \Delta u_i(x_1, x_2) = 0, \tag{2}$$

where $u = \{u_+, u_-, u_i\}$ denotes the displacement component along x_3-axis in the respective domain $\Omega_+ \setminus \{g_\varepsilon \cup \gamma_3^\varepsilon\}, \Omega_- \setminus \gamma_2^\varepsilon$ and g_ε.

We prescribe the following boundary conditions on the crack faces

$$\mu_\pm \frac{\partial u_\pm}{\partial x_2} = p_\pm \quad \text{on } \Gamma_\pm^\varepsilon \tag{3}$$

and the interface conditions

$$u_+ = u_-, \quad \mu_+ \frac{\partial u_+}{\partial x_2} = \mu_- \frac{\partial u_-}{\partial x_2} \quad \text{on } \Gamma^\varepsilon. \tag{4}$$

The transmission conditions for the elastic inclusion g_ε are formulated similarly to (4), that is,

$$u_+ = u_i, \quad \mu_+ \frac{\partial u_+}{\partial n} = \mu_i \frac{\partial u_i}{\partial n} \quad \text{on } \partial g_\varepsilon. \tag{5}$$

We assume that the microcrack faces $\gamma_2^\pm$ are traction free, so that

$$\frac{\partial u_-}{\partial n} = 0 \quad \text{on } \gamma_2^\pm. \tag{6}$$

Finally, on the boundary of the *movable* rigid line inclusion γ_3^ε the Dirichlet boundary condition is prescribed, that is,

$$u_+ = u_* \quad \text{on} \quad \gamma_3^\varepsilon, \tag{7}$$

where u_* is an unknown constant which will be defined later from the balance condition

$$\int_{\gamma_3^\varepsilon} \frac{\partial u_+}{\partial n} ds = 0. \tag{8}$$

The unperturbed problem ($\varepsilon = 0$) corresponds to a semi-infinite interfacial crack in a bimaterial plane. The solution to this problem